Impairment- and Splitting-Aware Cloud-Ready Multicast Provisioning in Elastic Optical Networks

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***Abstract*—It is known that multicast provisioning is important for supporting cloud-based applications, and as the traffics from these applications are increasing quickly, we may rely on optical networks to realize high-throughput multicast. Meanwhile, the flexible-grid elastic optical networks (EONs) achieve agile access to the massive bandwidth in optical fibers, and hence can provision variable bandwidths to adapt to the dynamic demands from cloud-based applications. In this paper, we consider all-optical multicast in EONs in a practical manner and focus on designing impairment- and splitting-aware multicast provisioning schemes. We first study the procedure of adaptive modulation selection for a light-tree, and point out that the multicast scheme in EONs is fundamentally different from that in the fixed-grid wavelength-division multiplexing (WDM) networks. Then, we formulate the problem of impairment- and splitting-aware routing, modulation and spectrum assignment (ISa-RMSA) for all-optical multicast in EONs and analyze its hardness. Next, we analyze the advantages brought by the flexibility of routing structures and discuss the ISa-RMSA schemes based on light-trees and light-forests. Our study suggests that for ISa-RMSA, the light-forest based approach can use less bandwidth than the light-tree based one, while still satisfying the quality of transmission (QoT) requirement. Therefore, we establish the minimum light-forest problem for optimizing a light-forest in ISa-RMSA. Finally, we design several time-efficient ISa-RMSA algorithms, and prove that one of them can solve the minimum light-forest problem with a fixed approximation ratio.**

***Index Terms*—Elastic optical networks (EONs), All-optical multicast, Routing, modulation and spectrum assignments (RMSA), Impairment, Approximation algorithm.**

I. Introduction

WITH the rapid deployment of bandwidth-hungry cloud-based applications, the traffic demand in Internet backbone and inter-datacenter networks has been increasing exponentially in the past few years. This trend has stimulated active research on the optical networking technologies that can facilitate highly scalable and flexible core networks. Recently, attributed to the advances on optical transmission and switching technologies, flexible-grid elastic optical networks (EONs) have been proposed to achieve efficient and agile access to the massive bandwidth in optical fibers [1, 2]. Specifically, EONs leverage bandwidth-variable transponders (BV-Ts) and switches (BV-WSS') to manage optical spectra with a fine granularity down to 12.5 GHz or less, and hence can establish lightpaths with variable bandwidths to adapt to the dynamic demands from cloud-based applications.

It is known that in EONs, each lightpath is provisioned with narrow-band subcarrier frequency slots (FS') that are spectrally contiguous, and the actual modulation-format used by each FS (*e.g.*, BPSK, QPSK, 8-QAM, and 16-QAM.) should be adaptively selected according to quality-of-transmission (QoT) [3]. Basically, as explained in [3], a higher modulation-level (*e.g.*, 16-QAM with respect to QPSK) provides higher spectral efficiency (*i.e.*, bits/s/Hz), and thus we can use fewer FS' (*i.e.*, less optical bandwidth) to accommodate the same capacity requirement with it. Meanwhile, due to physical impairments, a higher modulation-level usually results in reduced receiver sensitivity and thus only supports a shorter transmission reach. Therefore, different from the conventional fixed-grid wavelength-division multiplexing (WDM) networks that rely on routing and wavelength assignment (RWA) for service provisioning, EONs need to solve the problem of routing, modulation and spectrum assignment (RMSA) [4]. Apparently, RMSA in EONs is more complex than RWA and brings new challenges to network control and management (NC&M). Previous studies have considered RMSA in EONs, and proposed a few integer linear programming (ILP) models and heuristics to address it from different perspectives [1, 5–9]. However, these work did not consider the multicast provisioning for point-to-multiple-point communications.

Multicast provisioning is known to be important for supporting cloud-based applications, such as datacenter backup, grid computing, *etc*. Owing to the dynamic nature of these bandwidth-intensive applications, their traffics exhibit high throughput and high burstiness [2]. Fortunately, optical fibers can provide tremendous bandwidth and the technical advances on EONs have ensured that agile bandwidth management can be directly realized in the optical layer. Therefore, we expect EONs to provide not only effective but also reliable infrastructure to support cloud-based multicast applications. Although optical multicast [10] has been intensively investigated for WDM networks in literature, for schemes based on light-trees [11, 12] and optical-label switching [13–15], its provisioning schemes in EONs have just started to attract research interests recently [16–21]. Note that, with adaptive modulation selection [19], EONs can adjust the spectral usage of a light-tree[1]

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[1]Here, by saying a light-tree, we refer to the all-optical tree-type routing structure to carry a multicast request, which originates from the source and reaches all the destinations [11].

according to its QoT. Hence, the structure of a light-tree and the bandwidth assigned on it become correlated. This is fundamentally different from the case in WDM networks, where they are independent.

Moreover, if we consider the flexibility of routing structures in EONs and move one step forward, we can see that trying to provision a multicast request with only one light-tree may not be optimal or even practicable. Basically, due to the physical impairments from transmission and switch nodes, the light-tree may have to use the lowest modulation-level to ensure that all the destinations can receive the optical signal correctly. Consequently, the spectral efficiency would be the lowest and the light-tree can consume too much bandwidth. This is especially unwanted for bandwidth-hungry cloud-based applications, as the bandwidth resources can be drained away quickly to induce severe request blocking. Furthermore, for all-optical multicast, a single light-tree may be incapable to cover all the destination nodes due to QoT constraints. Hence, it would be more promising to serve the request with a light-forest that contains multiple light-trees because we can select higher modulation-levels to save the bandwidth usage while still satisfying the QoT requirement.

In this paper, we focus on designing impairment- and splitting-aware multicast provisioning schemes for EONs. We first formulate the mathematical problem of impairment- and splitting-aware RMSA (ISa-RMSA) with light-forests for all-optical multicast in EONs. The hardness of the problem is also analyzed. Then, we analyze the advantages brought by the flexibility of routing structures and discuss the ISa-RMSA schemes based on light-trees and light-forests. Our study shows that for ISa-RMSA, the light-forest based approach uses less bandwidth than the light-tree based one, while still satisfying the QoT requirement. Hence, we establish the minimum light-forest problem for optimizing a light-forest in ISa-RMSA. Finally, we design several time-efficient ISa-RMSA algorithms, and prove that one of them can solve the minimum light-forest problem with a fixed approximation ratio. Compared to the state of the art, the major contributions of this work include the following:

1) We formulate the problem of ISa-RMSA for all-optical multicast with a light-forest approach in EONs and analyze its complexity.
2) We show that light-tree outperforms light-forest in WDM networks, while a light-forest can achieve better multicast provisioning performance in terms of bandwidth utilization than a light-tree in EONs.
3) We establish the minimum light-forest problem to optimize a light-forest for ISa-RMSA, analyze its complexity, and prove that the problem is $\mathcal{APX}$-complete.
4) We design an approximation algorithm to solve the minimum light-forest problem for ISa-RMSA, and prove that it can provide solutions with a fixed approximation ratio.

The rest of the paper is organized as follows. Section II provides a survey on the related work. Then, we consider the QoT constraints from both transmission and switch nodes and describe the problem of ISa-RMSA in EONs in Section III. Section IV considers the flexibility of routing structures and discusses both light-tree and light-forest in ISa-RMSA. The time-efficient algorithms for ISa-RMSA are designed in Section V, and Section VI presents the performance evaluations with simulations. Finally, Section VII summarizes the paper.

II. Related work

For fixed-grid WDM networks, multicast provisioning has been studied intensively in literature. Sahasrabuddhe *et al.* [11] came up with the concept of light-tree to support multicast efficiently in IP-over-WDM networks. In order to avoid the high cost from optical-to-electrical-to-optical (O/E/O) conversions, the authors of [10] have studied all-optical multicast. The constrained multicast in WDM networks with sparse light-splitting has been considered in [12], where four algorithms were proposed to build light-trees. However, these studies did not consider the QoT degradation from optical switches. Basically, since light splitting in switches introduces power loss and extra noise, we cannot simply assume that a light-tree can support the same transmission reach as a lightpath.

For QoT-aware multicast provisioning in WDM networks, researchers have investigated K-drop light-tree in [22], under the assumption that a light-tree can only split the optical signal for at most K times due to QoT degradation. Yu and Cao [23] considered a more practical case in which each light-tree can only support a limited number of signal drops and the length of its longest source-destination branch is also bounded. Xin *et al.* [24] considered the power losses in the optical layer and proposed several algorithms to construct light-trees under physical constraints, while Ellinas *et al.* [25] studied the multicast routing algorithms based on Q-factor. For a complete review of multicast provisioning in WDM networks, readers are referred to the two surveys in [10, 26].

It is known that RMSA in EONs is more complex than RWA in WDM networks, and thus multicast provisioning needs to be revisited for EONs. Without the adaptive modulation selection, Wang *et al.* [16] analyzed the performance of two multicast-capable routing and spectrum assignment (MC-RSA) algorithms for EONs. In [18], we improved the performance of their schemes by leveraging a layered approach to design integrated MC-RSA algorithms. Meanwhile, we also incorporated adaptive modulation selection and studied multicast provisioning with impairment-aware RMSA in [19], where two integer linear programming (ILP) models as well as several heuristics were proposed. Later on, multicast provisioning with distance-adaptive transmission was studied in [20]. By considering the request scheduling in the time domain, we investigated the multicast provisioning with advance reservation in EONs in [27]. Walkowiak *et al.* [21] studied how the fanout of optical switches would affect the transmission reach of multicast signals in EONs. We also utilized light-forest with rateless network coding to design efficient all-optical multicast schemes for EONs in [28]. More recently, by leveraging the idea of software-defined EON (SD-EON) [29, 30], we experimentally demonstrated the control plane operation for provisioning multicast sessions in [31, 32].

Nevertheless, the flexibility of routing structures has not been fully explored for multicast provisioning with

TABLE I
MAPPING BETWEEN MODULATION FORMAT AND MAXIMUM TRANSMISSION REACH.

Modulation Format	Modulation-Level (m)	Transmission Reach (km)
BPSK	1	5000
QPSK	2	2500
8-QAM	3	1250
16-QAM	4	625

impairment- and splitting-aware modulation selection strategy. Therefore, in this work, we address the impairment- and splitting-aware multicast problem in EONs.

III. RMSA FOR ALL-OPTICAL MULTICAST IN EONS

A. Network Model

The EON's physical topology can be modeled as a directed graph $G(V, E)$, where V and E are the sets of the nodes and fiber links, respectively. Each link $e \in E$ accommodates a total bandwidth of B in GHz, while the optical spectrum is divided into narrow-band frequency slots (FS'), each of which has a bandwidth of $B_{\text{FS}} = 12.5$ GHz [33]. Hence, the total number of available FS' on an empty link is

$$F = \lfloor \frac{B}{B_{\text{FS}}} \rfloor. \tag{1}$$

A multicast request is denoted as $R(s, D, b)$, where $s \in V$ is the source node, $D \subseteq V \setminus s$ is the set of destinations, and b is the capacity requirement in Gb/s. We assume that the operator can select the modulation format for data transmission from BPSK, QPSK, 8-QAM and 16-QAM, according to the QoT, and assign the modulation-level as $m = 1$, 2, 3 and 4, respectively. Here, the modulation format and the maximum transmission reach M_m can be mapped with respect to Table I, if we only consider the impairments of unicast transmission, according to the experimental results in [3, 34]. Note that, we will discuss the optical signal-to-noise ratio (OSNR) degradation on multicast signals in optical switches and explain how the transmission reach would be affected later. Finally, with the assigned modulation-level m and capacity requirement b, we obtain the number of contiguous FS' to be assigned as [19]

$$n = \lceil \frac{b}{m \cdot C_{\text{BPSK}}} \rceil + g_b, \tag{2}$$

where C_{BPSK} is the capacity of an FS when using BPSK (*i.e.*, $m = 1$), and g_b represents the number of FS' that are used for the guard-band. We assume that $C_{\text{BPSK}} = 12.5$ Gb/s for the FS defined above and use $g_b = 1$ in this work.

In this work, we only consider the all-optical multicast scheme in which the spectrum assignment on a light-tree does not change end-to-end. This is because the all-optical spectrum conversion techniques for EONs are still not mature, while optical-to-electrical-to-optical (O/E/O) conversions incur a significant increase on equipment cost and power consumption. Hence, for a light-tree $\mathcal{T}$, we assign the same block of FS' on all the links, under the spectrum contiguous and continuity constraints [19]. It is known that to support all-optical multicast, a multicast-capable (MC) optical switch

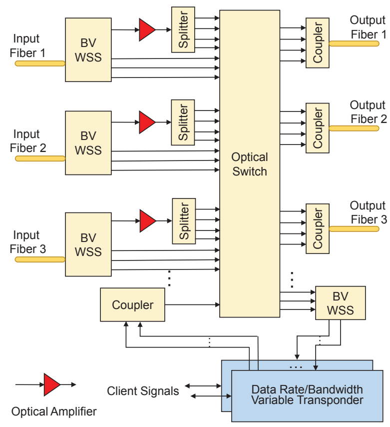


Fig. 1. Multicast-capable optical switch with the non-broadcast-and-select architecture (adapted from [38]).

has to implement light splitting, *i.e.*, splitting an input optical signal and delivering the copies to multiple output ports. In order to support this feature, several node architectures have been proposed, including the splitter-and-delivery (SaD) [35], the tap-and-continue (TaC) [36], the broadcast-and-select [37], the non-broadcast-and-select [38], and the passive drop-and-waste/filter-less architecture [9].

Here, we assume that all the nodes in $G(V, E)$ are MC optical switches. We conduct the analysis in this work based on the non-broadcast-and-select architecture proposed in [38] (as shown in Fig. 1). This is because it processes unicast and multicast signals separately, which helps to avoid unnecessary OSNR degradation on the unicast signals. Basically, as shown in Fig. 1, to implement light splitting, the MC optical switch forwards each multicast signal to a splitter while the unicast signals will bypass it. Then, the insertion loss of the splitter would make the power of the multicast signals significantly lower than that of the unicast signals. Hence, we insert an optical amplifier before each splitter to compensate for its power loss in advance (as shown in Fig. 1), such that the power-level difference between unicast and multicast signals at the input branches of the output couplers would be relatively small. However, the optical amplifiers will cause additional OSNR degradations to the multicast signals and thus the transmission reach of them would be reduced. Note that, the transmission reach reduction due to light splitting impacts the modulation selection strategy of multicast signals, and we leverage the approximation model developed in [21] to obtain the transmission reach of multicast signals as

$$\hat{M}_m = (1 - \alpha) \cdot M_m, \tag{3}$$

where $\hat{M}_m$ represents the maximum transmission reach for the multicast signal that uses modulation-level m. Basically, considering the possible extra OSNR degradation on multicast signals in MC optical switches, we assume that their transmission reaches, as compared to unicast signals, are reduced by a factor of α. Meanwhile, we hope to point out that this assumption remains to be confirmed by experimental data and the actual value of α should be determined according to the architecture of MC optical switches.

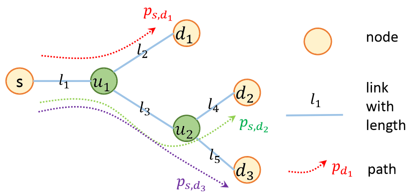


Fig. 2. Example of virtual length calculation for a light-tree.

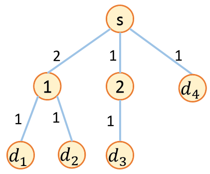


Fig. 3. Example of end-to-end flows in a light-tree.

B. Splitting-aware Modulation-Level Selection

Based on the aforementioned network model, we define a concept of "virtual length" to assist the splitting-aware modulation-level selection.

Definition 1. *The **virtual length** of a source-destination branch $p_{s,d}$ in light-tree $\mathcal{T}$ is defined as*

$$vlen(p_{s,d}) = \frac{1}{1-\alpha} \cdot len(p_{s,d}), \tag{4}$$

where $len(p_{s,d})$ is the actual physical length of branch $p_{s,d}$.

Definition 2. *The **virtual length** of light-tree $\mathcal{T}$ is defined as the maximum virtual length of all its branches*

$$vlen(\mathcal{T}) = \max_{d \in D}(vlen(p_{s,d})), \tag{5}$$

where D is the set of destinations in $\mathcal{T}$.

Fig. 2 gives an intuitive example on how to calculate the virtual length of a light-tree. Specifically, for the three destinations in the light-tree, the source-destination branches are p_{s,d_1}, p_{s,d_2} and p_{s,d_3}, and then by applying Eqs. (4) and (5) to the branches, we can get the virtual length of the light-tree. For instance, if we have $l_1+l_3+l_5 > l_1+l_3+l_4 > l_1+l_2$, which means that p_{s,d_3} has the longest physical length, we have $vlen(\mathcal{T}) = \frac{l_1+l_3+l_5}{1-\alpha}$. Then, for the splitting-aware modulation-level selection, we use Table I to map $vlen(\mathcal{T})$ to a proper modulation-level for the light-tree.

C. Problem Description of ISa-RMSA

In this work, we address ISa-RMSA for all-optical multicast in EONs for two scenarios, *i.e.*, static network planning and dynamic network provisioning.

In static network planning, the multicast requests are already known. Hence, we try to design an EON that can accommodate all the requests with the minimum spectrum resources. Usually, we can achieve this optimization objective by minimizing the maximum index of the used FS' (MSI) on any fiber link in the network. This is because in order to ensure the network-wide inter-operability, we need to allocate the same number of FS' on each fiber link $e \in E$ and hence MSI determines the total spectrum resources to be allocated in the network. On the other hand, dynamic network provisioning considers the scenario in which the EON is already built with a fixed number of FS' on each link, and we need to serve the dynamic multicast requests and make the best use of the spectrum resources. Hence, we need to design the provisioning scheme that can provide the smallest request blocking probability.

Talebi *et al.* [39] proved that for unicast in EONs, one of the sub-problems of RMSA, the spectrum assignment, is $\mathcal{NP}$-hard, even when the network topology is as simple as a single line with four or more links. Therefore, we can easily prove that ISa-RMSA for multicast is $\mathcal{NP}$-hard, because RMSA for unicast solves a special case of it when there is only one multicast destination. In Subsection IV-B, we will prove that ISa-RMSA for multicast in EONs, *i.e.*, the minimum light-forest problem, is $\mathcal{APX}$-complete. That is to say, it is $\mathcal{NP}$-hard to find an approximation algorithm for it, which can provide an approximation ratio less than a constant.

D. Mathematical Formulation

For static network planning, we formulate a flow-based mathematical model, which is inspired by the work in [20], to optimize ISa-RMSA for multicast.

Definition 3. *For a multicast light-tree $\mathcal{T}$, we define an **end-to-end flow** as the connection from the source to a destination.*

For instance, Fig. 3 shows an example. The light-tree contains 4 destinations, *i.e.*, d_1, d_2, d_3 and d_4, and *Nodes* 1 and 2 are the intermediate nodes. Since there is a light-splitting on *Node* 1, $s \to 1 \to d_1$ and $s \to 1 \to d_2$ become two end-to-end flows. Hence, the number of flows on *Link* $s \to 1$ is 2. Similarly, we can get the numbers of flows on the other links as shown in Fig. 3.

Tables II and III list the notations and variables in the mathematical formulation, respectively. The optimization objective of the ISa-RMSA problem is defined as follows.

Objective:

$$Minimize \quad M \cdot msi + \sum_{i,k,e} y_{i,k,e}. \tag{6}$$

Here, the major optimization objective (*i.e.*, the first term) is to minimize MSI, as it represents the minimum amount of spectrum resources that we should provide on each fiber link to serve all the multicast requests. The second term is introduced to avoid unnecessary spectrum utilization when MSI is the same, *i.e.*, to minimize the total FS usage.

Constraints:

1) *Flow-related Constraints:*

$$\sum_{v \in V} f_{i,k,(v,s_i)} = 0, \quad \forall R_i \in \mathbf{R}, k \in [1, T_{max}]. \tag{7}$$

Eq. (7) ensures that the number of input flows to the source of each request is 0 in its light-forest.

$$\sum_{v \in V} f_{i,k,(s_i,v)} = |D_i|, \quad \forall i, k. \tag{8}$$

TABLE II
PARAMETERS IN MATHEMATICAL FORMULATION OF ISA-RMSA PROBLEM

Notations	Definition
$G(V,E)$	the network topology, where V and E are the sets of nodes and links, respectively.
$e=(v,u)$	the link $e \in E$ that is from v to u, where v and u are adjacent nodes and $v, u \in V$.
$L_{(u,v)}$	the length of link $(u,v) \in E$ in kilometers.
$\mathbf{R}$	the set of multicast requests, and each element is denoted as $R_i=(s_i,D_i,b_i)$, where i is the request index.
C_{BPSK}	the capacity of an FS when using BPSK.
g_b	the number of FS' that are used for the guard-band of each light-tree.
$\mathbf{M_0}$	the mapping between unicast transmission reach and modulation-level, as shown in Table I.
M_m	the unicast transmission reach of modulation-level m, *i.e.*, $M_m \in \mathbf{M_0}$.
$\hat{M}_m$	the multicast transmission reach of modulation-level m, which is calculated with Eq. (3).
α	the transmission reach reduction ratio due to all-optical multicast.
M	a large constant number that is larger than the upper bound of the number of FS' on each link.
T_{max}	the maximum number of light-trees, *i.e.*, a light-forest cannot contain more than T_{max} light-trees.

TABLE III
VARIABLES IN MATHEMATICAL FORMULATION OF ISA-RMSA PROBLEM

Variables	Definition
$f_{i,k,e}$	the integer variable that represents the number of flows on link $e \in E$ in the k-th light-tree of request R_i.
$y_{i,k,e}$	the boolean variable that equals 1 if link $e \in E$ is used by the k-th light-tree of request R_i, and 0 otherwise.
$h_{i,k,d}$	the boolean variable that equals 1 if destination $d \in D_i$ is served in the k-th light-tree, and 0 otherwise.
$vlen_{i,k,v}$	the real variable that indicates the virtual length of the sub-light-tree that roots at node v in the k-th light-tree of R_i.
$w_{i,k}$	the integer variable that represents the start-index of the assigned FS-block on the k-th light-tree of R_i.
$z_{i,k}$	the integer variable that represents the end-index of the assigned FS-block on the k-th light-tree of R_i.
$m_{i,k,m}$	the boolean variable that equals 1 if modulation-level m can be used to serve the k-th light-tree of R_i, and 0 otherwise.
$m_{i,k}$	the integer variable that represents the modulation-level of the k-th light-tree of R_i.
$n_{i,k}$	the integer variable that represents the number of FS' that needs to be allocated on the k-th light-tree of R_i.
c_{i,j,k_1,k_2}	the boolean variable that equals 1 if the k_1-th light-tree of R_i and the k_2-th light-tree of R_j use at least one common link, and 0 otherwise.
c_{i,k_1,k_2}	the boolean variable that equals 1 if the k_1-th light-tree and the k_2-th light-tree of R_i use at least one common link, and 0 otherwise, $k_1 \neq k_2$.
o_{i,j,k_1,k_2}	the boolean variable that equals 1 if $w_{i,k_1} < w_{j,k_2}$, and 0 otherwise.
o_{i,k_1,k_2}	the boolean variable that equals 1 if $w_{i,k_1} < w_{i,k_2}$, and 0 otherwise, $k_1 \neq k_2$.
msi	the integer variable that represents the maximum index of the used FS' (MSI) in the network.

Eq. (8) ensures that the number of output flows from each source equals to the number of destinations, *i.e.*, $|D_i|$.

$$\sum_{u \in V} f_{i,k,(u,v)} = \sum_{u \in V} f_{i,k,(v,u)}, \ \forall i,k, \forall v \in V \setminus \{s_i, D_i\}. \quad (9)$$

Eq. (9) ensures that on each intermediate node on the light-trees, the numbers of input and output flows are equal.

$$\sum_{u \in V} f_{i,k,(u,d)} = \sum_{u \in V} f_{i,k,(d,u)} + h_{i,k,d}, \ \forall i,k, \ \forall d \in D_i. \quad (10)$$

Eq. (10) ensures that on each destination on the light-trees, the number of output flows is one less than that of input flows.

$$\sum_{k} h_{i,k,d} = 1, \quad \forall i, \ \forall d \in D_i. \quad (11)$$

Eq. (11) ensures that for each request R_i, a destination should be served in one and only one light-tree.

2) *Length-related Constraints*:

$$vlen_{i,k,d} = 0, \quad \forall i,k, \ \forall d \in D_i. \quad (12)$$

Eq. (12) ensures that the virtual length of the sub-light-tree that rooted at a destination is 0.

$$\begin{aligned} vlen_{i,k,v} - vlen_{i,k,u} \geq \frac{L_{(v,u)}}{1-\alpha} - M \cdot (1 - y_{i,k,(v,u)}), \\ \forall i,k, \ \forall (v,u) \in E. \end{aligned} \quad (13)$$

Eq. (13) determines the virtual length of the sub-light-tree rooted at $v \in V$ on the k-th light-tree of R_i.

3) *Modulation-Level Selection Constraints:*

$$m_{i,k,m} \cdot vlen_{i,k,s_i} \leq \hat{M}_m, \quad \forall i,k,m. \quad (14)$$

Eq. (14) ensures that if modulation-level m can be used on the k-th light-tree of R_i, its unicast transmission reach should not be shorter than the virtual length of the light-tree.

$$m_{i,k} \leq \sum_{m} m_{i,k,m}, \quad \forall i,k. \quad (15)$$

Eq. (15) gets the modulation-level of the k-th light-tree of R_i.

$$n_{i,k} \geq \lceil \frac{b_i}{C_{\text{BPSK}} \cdot m_{i,k}} \rceil + g_b, \quad \forall i,k. \quad (16)$$

Eq. (16) determines the number of FS' to be allocated on the k-th light-tree of R_i.

4) *Spectrum Assignment Constraints:*

$$f_{i,k,e} \leq M \cdot y_{i,k,e}, \quad \forall i,k,e. \quad (17)$$

Eq. (17) determines the relation between the number of flows and the link usage for each light-tree.

$$\begin{aligned} c_{i,k_1,k_2} \geq y_{i,k_1,e} + y_{i,k_2,e} - 1, \quad \forall i,e, \\ \{k_1,k_2 \in [1,T_{max}] : k_1 \neq k_2\}. \end{aligned} \quad (18)$$

Eq. (18) ensures that c_{i,k_1,k_2} is determined correctly.

$$\begin{aligned} c_{i,j,k_1,k_2} \geq y_{i,k_1,e} + y_{j,k_2,e} - 1, \\ \forall k_1,k_2,e,\{R_i,R_j : i \neq j\}. \end{aligned} \quad (19)$$

Eq. (19) ensures that c_{i,j,k_1,k_2} is determined correctly.

$$o_{i,k_1,k_2} + o_{i,k_2,k_1} = 1. \ \forall i, \{k_1, k_2 : k_1 \neq k_2\}. \tag{20}$$

Eq. (20) ensures that values of o_{i,k_1,k_2} and o_{i,k_2,k_1} correctly represent the relation between w_{i,k_1} and w_{i,k_2} based on the FS-blocks assigned on the k_1-th and k_2-th light-trees of R_i.

$$o_{i,j,k_1,k_2} + o_{i,j,k_2,k_1} = 1, \ \forall k_1, k_2, \{R_i, R_j : i \neq j\}. \tag{21}$$

Eq. (21) ensures that values of o_{i,j,k_1,k_2} and o_{i,j,k_2,k_1} correctly represent the relation between w_{i,k_1} and w_{j,k_2} based on the FS-blocks assigned on the k_1-th light-tree of R_i and the k_2-th light-tree of R_j.

$$z_{i,k} - w_{i,k} + 1 \geq n_{i,k} \cdot y_{i,k,e}, \quad \forall i, e, k. \tag{22}$$

Eq. (22) ensures that the FS' assigned to each multicast request can satisfy its capacity requirement.

$$\begin{gathered} z_{i,k_2} - w_{i,k_1} + 1 \leq M \cdot (1 + o_{i,k_1,k_2} - c_{i,k_1,k_2}), \\ \forall i, \{k_1, k_2 : k_1 \neq k_2\}. \end{gathered} \tag{23}$$

$$\begin{gathered} z_{i,k_1} - w_{i,k_2} + 1 \leq M \cdot (2 - o_{i,k_1,k_2} - c_{i,k_1,k_2}), \\ \forall i, \{k_1, k_2 : k_1 \neq k_2\}. \end{gathered} \tag{24}$$

$$\begin{gathered} z_{j,k_2} - w_{i,k_1} + 1 \leq M \cdot (1 + o_{i,j,k_1,k_2} - c_{i,j,k_1,k_2}), \\ \forall k_1, k_2, \{R_i, R_j : i \neq j\}. \end{gathered} \tag{25}$$

$$\begin{gathered} z_{i,k_1} - w_{j,k_2} + 1 \leq M \cdot (2 - o_{i,j,k_1,k_2} - c_{i,j,k_1,k_2}), \\ \forall k_1, k_2, \{R_i, R_j : i \neq j\}. \end{gathered} \tag{26}$$

Eqs. (23)-(26) ensure that all the spectrum assignments satisfy the spectrum non-overlapping constraint.

5) *MSI-related Constraint*:

$$msi \geq z_{i,k}, \forall i, k. \tag{27}$$

Eq. (27) determines MSI in the network.

It can be seen that the problem formulation discussed above is not a mixed integer linear programming (MILP) model because the constraints in Eqs. (14), (16) and (22) are not linear. Meanwhile, considering the fact that the scale of the model, *i.e.*, the number of variables and constraints in it, is relatively large, it would be a complex problem to solve.

IV. Light-Tree versus Light-Forest in ISa-RMSA

Since the mathematical model in the previous section is nonlinear and cannot be solved in a time-efficient manner, we need to investigate the ISa-RMSA problem further. Note that, for multicast provisioning in EONs, the routing structure to cover a multicast request and the bandwidth assigned on each link in it are correlated. This is fundamentally different from the multicast provisioning in WDM networks. Hence, we can see that the routing subproblem in ISa-RMSA is essential and should be considered carefully for achieving good multicast provisioning performance. In this section, we try to discuss the routing structure to minimize the number of FS' used for serving a multicast request.

A. Light-Forest Structure for Multicast Provisioning

The discussion in Subsection III-B suggests that the modulation-level and thus the spectral efficiency of a light-tree depends on the branch whose virtual length is the longest. Hence, if a light-tree has significantly unbalanced branches, *i.e.*, their virtual lengths vary a lot, the spectral efficiency of the branches that have shorter virtual lengths would be brought down by those with longer virtual lengths. Therefore, to save spectrum resources, we need to balance the light-tree's branches such that they have similar virtual lengths. If certain branches cannot be balanced, we leverage the light-forest scheme and split the light-tree into multiple smaller ones. Then, those branches can be isolated in small light-trees, and thus the low spectral efficiency is only used to serve a small number of destinations. Consequently, the total FS usage is reduced. Hence, to serve a multicast request $R(s, D, b)$, a light-forest might be a better routing structure than a light-tree.

Definition 4. *A **light-forest** $\mathcal{F}$ consists of several light-trees $\{\mathcal{T}_1, \mathcal{T}_2, ..., \mathcal{T}_k\}$, and the i-th light-tree covers a set of destinations $D_i \in D$. Each destination must and can only be covered by one of the light-trees,* i.e., *$D_1 \bigcup D_2 \bigcup ... \bigcup D_k = D$ while $D_i \bigcap D_j = \Phi, \ \{i, j : i \neq j\}$.*

Definition 5. *The **cost of a light-forest** $\mathcal{F}$ is defined as*

$$c(\mathcal{F}) = \sum_{i \leq k} c(\mathcal{T}_i) = \sum_{i \leq k} n_i \cdot h_i, \tag{28}$$

where $c(\mathcal{T}_i)$ is the cost or bandwidth consumption of the i-th light-tree in terms of FS', and n_i and h_i are the number of used FS' per link and the light-tree's hop-count, respectively.

In the light-forest for a request $R(s, D, b)$, each light-tree chooses its modulation-level independently. If the i-th light-tree chooses modulation-level m_i and its hop-count is h_i, the bandwidth consumption is $c(\mathcal{T}_i) = \left(\lceil \frac{b}{m_i \cdot C_{\text{BPSK}}} \rceil + g_b\right) \cdot h_i$. Hence, the cost of the light-forest is calculated as

$$c(\mathcal{F}) = \sum_{i \leq k} c(\mathcal{T}_i) = \sum_{i \leq k} \left(\lceil \frac{b}{m_i \cdot C_{\text{BPSK}}} \rceil + g_b\right) \cdot h_i. \tag{29}$$

Definition 6. *For a multicast request, a **minimum light-forest** $\mathcal{F}_{\min}$ is the light-forest that has the minimum cost $c(\mathcal{F})$ among all the possible light-forests.*

B. Hardness Analysis on Minimum Light-Forest Problem

For a multicast request $R(s, D, b)$, the minimum light-forest $\mathcal{F}^*$ consists of a set of light-trees $\{\mathcal{T}_1, \cdots, \mathcal{T}_i, \cdots\}$ such that they cover all the destinations in D and their total cost $\sum\limits_i c(\mathcal{T}_i)$ is the minimum.

Theorem 1. *The minimum light-forest problem in EONs is $\mathcal{APX}$-complete.*

Proof: We first prove that a special case of the minimum light-forest problem is the steiner-tree problem. We consider a special case in which the multicast request $R(s, D, b)$ only demands for such a small amount of transmission capacity that we should only allocate one FS on each link in the light-forest, no matter what modulation-level is chosen. For this special case, the minimum light-forest problem becomes to find the light-tree that contains the smallest number of links to connect all the nodes in $\{s, D\}$. Hence, we reduce the special case of the minimum light-forest problem to the steiner-tree problem.

Meanwhile, it is known that this version of steiner-tree problem is $\mathcal{APX}$-complete, *i.e.*, there does not exist an approximation algorithm with the approximation ratio that is less than a constant [40]. The latest study in [41] proved that the constant should be $\frac{96}{95}$. Therefore, we prove that the minimum light-forest problem is at least $\mathcal{APX}$-complete. ■

C. Light-Tree versus Light-Forest

A light-forest uses several light-trees to cover all the destinations of a multicast request, and has been previously used in WDM networks to satisfy the QoT constraints that a single light-tree cannot support [22]. However, since flexible bandwidth allocation and adaptive modulation selection are not feasible in fixed-grid WDM networks, the bandwidth consumption per link for a request is fixed. Therefore, the minimum light-forest problem is reduced to the steiner tree problem, and for a multicast request, the light-forest structure cannot obtain any benefit in terms of bandwidth consumption as compared with the light-tree structure. However, with the adaptive modulation selection in EONs, different modulation-levels lead to various FS usages, thus possibly making the light-forest structure be more spectrum efficient.

Theorem 2. *When serving a multicast request in a WDM network, the optimal light-tree (i.e., the light-tree that consumes the least bandwidth) does not consume more bandwidth resources than any light-forest.*

Proof: We prove the theorem by contradiction. For a given multicast request, we denote the bandwidth consumption of the optimal light-tree $\mathcal{T}^*$ as $c(\mathcal{T}^*)$, while the bandwidth consumption of the optimal light-forest $\mathcal{F}^*$ is $c(\mathcal{F}^*)$.[2]

We first assume that $c(\mathcal{T}^*) > c(\mathcal{F}^*)$. Then, we can construct a new light-tree $\mathcal{T}'$ from the optimal light-forest $\mathcal{F}^*$ by merging all the source nodes of its light-trees together. Since the bandwidth assigned on a light-tree is independent of its structure in a WDM network, the merging of source nodes will not change the bandwidth consumption on each link in $\mathcal{F}^*$. Considering the fact that we may be able to further merge certain links in $\mathcal{F}^*$ after merging the source nodes, we can obtain the bandwidth consumption of the new light-tree $\mathcal{T}'$ as $c(\mathcal{T}') \leq c(\mathcal{F}^*)$. Consequently, we have $c(\mathcal{T}') \leq c(\mathcal{F}^*) < c(\mathcal{T}^*)$ or $c(\mathcal{T}') < c(\mathcal{T}^*)$, implying that $\mathcal{T}'$ consumes less bandwidth than $\mathcal{T}^*$. This, however, contradicts with the fact that $\mathcal{T}^*$ is the optimal light-tree for the request. Hence, we prove that when serving a multicast request in a WDM network, the optimal light-tree does not consume more bandwidth resources than any light-forests. ■

However, *Theorem* 2 is not valid for the multicast in EONs because the adaptive modulation selection makes the structure of a light-tree and the bandwidth assigned on each link on it

[2]In this work, we define the bandwidth consumption of a light-forest as the summation of the bandwidths assigned on all the links in it. This definition also applies to a light-tree, as it is a special case of a light-forest.

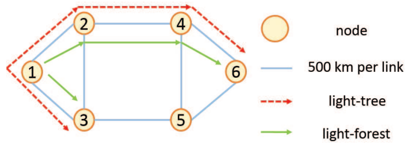


Fig. 4. Serving a multicast request with a light-tree and a light-forest.

be correlated. Specifically, as the light-trees in the light-forest have shorter virtual lengths than the one that covers the whole request, we can use higher modulation-levels on them for improved spectral efficiency. Hence, the optimal light-forest can consume less bandwidth than the optimal light-tree.

For instance, for the request $R(1, \{2, 3, 4, 6\}, 100\ Gb/s)$ in the six-node topology in Fig. 4, we can serve it with either a light-tree or a light-forest. The adaptive modulation selection determines the modulation-level for a light-tree based on its virtual length. If we assume $\alpha = 0.2$, then the optimal light-tree in Fig. 4 can only use QPSK as its virtual length is 1875 km, while the two small light-trees in the light-forest have their virtual lengths as 1875 km and 625 km, respectively. This means that the two small light-trees can use QPSK and 16-QAM as their modulation-levels. Hence, if we assume $g_b = 1$, the bandwidth consumptions of the light-tree and the light-forest are $4 \cdot (\lceil \frac{100}{12.5*2} \rceil + 1) = 20$ FS' and $3 \cdot (\lceil \frac{100}{12.5*2} \rceil + 1) + 1 \cdot (\lceil \frac{100}{12.5*4} \rceil + 1) = 18$ FS', respectively. The light-forest consumes 2 FS' less than the light-tree at the price of using one additional BV-T in the source node. Moreover, since the small light-trees in the light-forest may require not only less links but also smaller numbers of FS' per link, they would be accommodated in the EON more easily.

V. ISa-RMSA Algorithms

In this section, we design several time-efficient ISa-RMSA algorithms to solve the minimum light-forest problem by obtaining light-forests with balanced light-trees. We also consider spectrum fragmentation in dynamic provisioning.

A. Node-based Light-Tree Decomposition and Pruning

Our idea is to obtain an optimal light-tree without considering the QoT constraints first and then modify it to a feasible light-forest that satisfies the QoT constraints. *Algorithm* 1 shows the detailed procedure to realize the light-forest construction with the node-based light-tree decomposition and pruning (N-LT-DP) scheme.

In order to modify a pre-calculated optimal light-tree $\mathcal{T}$ for $R(s, D, b)$[3], *Lines* 1-4 first calculate the lengths of all the source-destination branches. Then, *Lines* 5-25 release destination d on the branch that has the longest virtual length and try to insert d into one of the existing light-trees. Specifically, on each existing light-tree, we select the branch that has the shortest virtual length to accept d as shown in *Lines* 8-17. *Lines* 10-16 check whether d can be inserted into one of the existing light-trees while still making the light-tree satisfy the

[3]The optimal light-tree can be obtained as the minimum-spanning tree (MST) or the shortest-path tree (SPT) to cover $\{s, D\}$ in $G(V, E)$

QoT constraint of a modulation-level. Note that, we also build a new light-tree to connect s and d with the minimum hop-count, and pre-add it to the light-forest as a new one as shown in *Line* 18. Then, we insert d into one of the existing light-trees in the light-forest such that the incremental consumption on FS' is minimized. *Lines* 22-24 indicate that the light-tree modification stops when the original light-tree $\mathcal{T}_1$ becomes a feasible one under the QoT constraints. Finally, the original light-tree is modified to a light-forest containing light-trees that satisfy the QoT constraints.

Algorithm 1: Node-based Light-Tree Decomposition and Pruning Algorithm (N-LT-DP)

---

input : Physical topology $G(V, E)$, multicast request $R(s, D, b)$ and pre-calculated light-tree $\mathcal{T}$.
output: Light-forest $\mathcal{F}$, modulation-level m_k for each light-tree $\mathcal{T}_k \in \mathcal{F}$.

1 **for** *all source-destination branches in* $\mathcal{T}$ **do**
2   calculate its length;
3 **end**
4 $flag = 1$, $\mathcal{T}_1 = \mathcal{T}$, $\mathcal{F} = \{\mathcal{T}_1\}$;
5 **while** $flag = 1$ **do**
6   choose the source-destination branch p_{max} that has the longest virtual length in $\mathcal{T}_1$;
7   modify p_{max} to remove its destination d;
8   **for** *each existing light-tree* $\mathcal{T}_i$ *in* $\mathcal{F}$ **do**
9   choose the source-destination branch p_{min} that has the shortest virtual length in $\mathcal{T}_i$;
10   **for** *all the nodes* $v \in p_{min}$ **do**
11   pre-add d in $\mathcal{T}_i$ by connecting it to v;
12   **if** $\mathcal{T}_i$ *is a feasible light-tree that can be served with a modulation-level* m_i **then**
13   calculate the incremental cost of $\mathcal{T}_i$ after pre-addition;
14   **break;**
15   **end**
16   **end**
17   **end**
18   find a new light-tree to connect s and d with the minimum hop and pre-insert it into set $\{\mathcal{T}_i\}$;
19   choose the light-tree $\mathcal{T}_i$ that has the minimum cost in Eq. (28) after pre-addition;
20   add d to $\mathcal{T}_i$ and update its modulation-level m_i;
21   update $\mathcal{F}$;
22   **if** $\mathcal{T}_1$ *becomes a feasible light-tree* **then**
23         $flag = 0$;
24   **end**
25 **end**

The time complexity of N-LT-DP contains two parts. The first one is the complexity of pre-calculating a light-tree, which is $O((|D|+1)\cdot|V|^2)$ according to [42]. The second part is for the light-tree decomposition and pruning, and the complexity depends on the while-loop (*i.e.*, *Lines* 5-25 in *Algorithm* 1). In the worst case, the while-loop will run $|D|$ times. There are two for-loops in the while-loop. The operation number of the first one is bounded by the number of light-trees in the light forest, which is $|D|-1$ in the worst case. The running times of the second for-loop are bounded by the longest path in the network, which cannot exceed the number of nodes in the topology, *i.e.*, $|V|-1$. In the second for-loop, the complexity of re-calculating the virtual length is $(|D|-1)\cdot(|V|-1)$. Hence, the total complexity of N-LT-DP is $O(|D|\cdot(|D|-1)\cdot(|V|-1)\cdot(|D|-1)\cdot(|V|-1)) = O(|D|^3\cdot|V|^2)$. Note that, as we pre-calculate the shortest path between each node pair in the topology, the complexity of path computation is ignored.

B. Node-based Dynamic Light-Forest Construction

The second algorithm is the node-based dynamic light-forest construction (N-DLFC), which builds the light-forest in iterations directly, without a pre-calculated light-tree. In each iteration, the algorithm tries to obtain light-trees with the most balanced branches. Specifically, N-DLFC adds one destination $d \in D$ in the light-forest in each iteration, in ascending order of the length of the shortest path between s and d, until all the destinations in D are covered by the light-forest. The node-addition operation of N-DLFC reuses *Lines* 8-21 in *Algorithm* 1. As compared with N-LT-DP, N-DLFC does not pre-calculate a light-tree, and hence its complexity is the same as that of the second part of N-LT-DP, which is $O(|D|^3\cdot|V|^2)$.

C. Branch-based Light-Tree Decomposition and Pruning

Since the node-based algorithms process one destination at a time without considering the relation among the destinations, they may not be efficient for certain multicast requests. Therefore, we propose to modify the pre-calculated light-tree with a branch-based approach. *Algorithm* 2 shows the details of the branch-based light-tree decomposition and pruning (B-LT-DP). Here, instead of releasing and re-inserting the destinations one by one, B-LT-DP deletes a whole branch from the pre-calculated light-tree each time as shown in *Lines* 5-16.

Although B-LT-DP handles a branch of destinations each time, each destination in a branch needs to be modified by N-LT-DP in the worst case. If we denote the numbers of destinations in the light-trees as $|D_1|, |D_2|, \ldots, |D_i|$, where $|D_1| + |D_2| + \ldots + |D_i| = |D|$, the complexity of B-LT-DP is $O((|D_1|^3 + |D_2|^3 + \ldots + |D_i|^3)\cdot|V|^2) \leq O(|D|^3\cdot|V|^2)$. In the extreme case when there is only one light-tree in the forest, the complexity is $O(|D|^3\cdot|V|^2)$.

D. Fragmentation-Aware Algorithm Design

Even though the aforementioned N-LF-DP, N-DLFC and B-LT-DP can solve the minimum light-forest problem for static network planning, their performance can still be improved further in dynamic network provisioning. This is because dynamic network provisioning can generate small and isolated spectrum fragments that are hard to be used by future requests, and thus can degrade the network performance significantly [43]. This issue can be relieved by incorporating fragmentation-awareness in the proposed algorithms. Specifically, we consider the fragmentation ratios of links when

Algorithm 2: Branch-based Light-Tree Decomposition and Pruning Algorithm (B-LT-DP)

input : Physical topology $G(V, E)$, multicast request $R(s, D, b)$ and pre-calculated light-tree $\mathcal{T}$.
output: Light-forest $\mathcal{F}$, modulation-level m_k for each light-tree $\mathcal{T}_k \in \mathcal{F}$.

for *all the source-destination branches in* $\mathcal{T}$ **do**
  calculate its length;
end
$flag = 1$, $\mathcal{T}_1 = \mathcal{T}$, $\mathcal{F} = \{\mathcal{T}_1\}$;
while $flag = 1$ **do**
  choose the source-destination branch that covers the smallest number of destinations in $\mathcal{T}_1$;
  remove the branch from $\mathcal{T}_1$;
  set up a new light-tree to connect all the destinations on the branch to s;
  if *the branch is not feasible* **then**
    modify the branch with *Algorithm* 1;
  end
  update $\mathcal{F}$;
  if $\mathcal{T}_1$ *becomes a feasible light-tree* **then**
    $flag = 0$;
  end
end

building the light-forest in N-LF-DP, N-DLFC and B-LT-DP. The fragmentation ratio is defined as [44]

$$\eta_e = 1 - \frac{1}{n_e}, \tag{30}$$

where n_e is the number of available FS-blocks on e. Then, we mark the weighted length of link e as $(1 + \eta_e) \cdot L_e$. Because the fragmentation ratio of a link can change during dynamic operation, we cannot just pre-calculate the shortest paths between all the node pairs, and the weighted shortest paths have to be re-calculated for each request. The complexity of this procedure is $O(|V|^2 \cdot (|E| + |V| \cdot log|V|))$, *i.e.*, we run the Dijkstra's algorithm ($O((|E| + |V| \cdot log|V|))$ [45]) for $|V|^2$ times to get the weighted shortest paths between all the node pairs. The complexity of calculating the fragmentation ratio is $O(|E| \cdot F)$, where F is the number of FS' on each link. Finally, the complexity of the proposed fragmentation-aware algorithms are $O(|V|^2 \cdot (|E| + |V| \cdot log|V| + |D|^3))$.

E. Approximation Ratio

We have proved that the minimum light-forest problem is $\mathcal{APX}$-hard. In this subsection, we will show that N-DLFC is an approximation algorithm for it with an approximation ratio of $|D| \cdot M_{gap}$. Here, for all the feasible modulation-levels $\{m\}$ in the EON, we have $M_{gap} = \frac{\max(m)}{\min(m)}$.

Theorem 3. *N-DLFC is an approximation algorithm for the minimum light-forest problem with an approximation ratio of* $|D| \cdot M_{gap}$.

Proof: We assume that the final light-forest constructed by N-DLFC for multicast request $R(s, D, b)$ is $\mathcal{F}$, which contains k light-trees $\{\mathcal{T}_1, \mathcal{T}_2, ..., \mathcal{T}_k\}$. m_i and h_i are the modulation-level and hop-count of the i-th light-tree $\mathcal{T}_i$. Light-tree $\mathcal{T}_i$ consumes $(\lceil \frac{b}{m_i \cdot C_{\text{BPSK}}} \rceil + g_b)$ FS' per link, and the total cost of light-forest $\mathcal{F}$ can be obtained as

$$c(\mathcal{F}) = \sum_{i \le k} \left(\lceil \frac{b}{m_i \cdot C_{\text{BPSK}}} \rceil + g_b \right) \cdot h_i. \tag{31}$$

As b and C_{BPSK} are constants, we define $C_0 = \frac{b}{C_{\text{BPSK}}}$ and have

$$c(\mathcal{F}) = \sum_{i \le k} \left(\lceil \frac{C_0}{m_i} \rceil + g_b \right) \cdot h_i. \tag{32}$$

Then, we start our analysis by considering the special case in which $R(s, D, b)$ is served with a light-forest $\mathcal{F}'$ whose light-trees are all unicast lightpaths. Hence, $\mathcal{F}'$ contains $|D|$ light-trees and its total cost is

$$c(\mathcal{F}') = \sum_{i \le |D|} \left(\lceil \frac{C_0}{m'_i} \rceil + g_b \right) \cdot h'_i, \tag{33}$$

where m'_i and h'_i are the modulation-level and hop-count of light-tree $\mathcal{T}'_i$, where $\mathcal{T}'_i \in \{\mathcal{T}'_1, ..., \mathcal{T}'_{|D|}\}$. In this special case, each light-tree, *i.e.*, the unicast lightpath that connects the source s to a destination $d \in D$, requires the minimum cost. Hence, it is obvious that the maximum cost of these light-trees will not be greater than that of the minimum light-forest, *i.e.*,

$$\max_i \left[\left(\lceil \frac{C_0}{m'_i} \rceil + g_b \right) \cdot h'_i \right] \le c(\mathcal{F}_{\text{OPT}}). \tag{34}$$

This is because $\left[\left(\lceil \frac{C_0}{m'_i} \rceil + g_b \right) \cdot h'_i \right]$ is the minimum cost to reach the corresponding destination and the light-forest $\mathcal{F}'$ has to cover all the destinations. Therefore, we have

$$\begin{aligned} c(\mathcal{F}') &= \sum_{i \le |D|} \left(\lceil \frac{C_0}{m'_i} \rceil + g_b \right) \cdot h'_i \\ &\le |D| \cdot \max_i \left[\left(\lceil \frac{C_0}{m'_i} \rceil + g_b \right) \cdot h'_i \right] \\ &\le |D| \cdot c(\mathcal{F}_{\text{OPT}}). \end{aligned} \tag{35}$$

Note that, in the light-forest construction of N-DLFC, the path between two nodes is calculated as the one with the minimum hop-count. Hence, if we assume that N-DLFC first builds an initial light-forest $\mathcal{F}''$ that sets up a unicast lightpath for each destination, the hop-count of each lightpath would be $h''_i \le h'_i$. Since we have $m''_i \cdot M_{gap} \ge m'_i$, which leads to

$$\lceil \frac{1}{m''_i} \rceil \cdot h''_i \le M_{gap} \cdot \lceil \frac{1}{m'_i} \rceil \cdot h'_i, \tag{36}$$

the cost of the light-forest will be

$$\begin{aligned} c(\mathcal{F}'') &= \sum_{i \le |D|} \left(\lceil \frac{C_0}{m''_i} \rceil + g_b \right) \cdot h''_i \\ &\le M_{gap} \cdot \sum_{i \le |D|} \left(\lceil \frac{C_0}{m'_i} \rceil + g_b \right) \cdot h'_i \\ &\le M_{gap} \cdot |D| \cdot c(\mathcal{F}_{\text{OPT}}). \end{aligned} \tag{37}$$

Then, N-DLFC can only insert a destination into an existing light-tree $\mathcal{T}_i$ when we have

$$c(\hat{\mathcal{T}}_i) \le c(\mathcal{T}_i) + c(\mathcal{T}_{s,d}), \tag{38}$$

where $\hat{\mathcal{T}}_i$ is the light-tree with d inserted, and $\mathcal{T}_{s,d}$ is the lightpath that connects s and d with the minimum cost. This means that in each operation, we ensure that the incremental consumption on FS' is minimized. Therefore, we guarantee that after N-DLFC having inserted all the destinations, the cost of the final obtained light-forest $\mathcal{F}$ should satisfy

$$\begin{aligned} c(\mathcal{F}) &= \sum_{i\leq k}\left(\lceil\frac{C_0}{m_i}\rceil + g_b\right)\cdot h_i \\ &\leq c(\mathcal{F}'') = \sum_{i\leq|D|}\left(\lceil\frac{C_0}{m''_i}\rceil + g_b\right)\cdot h''_i \\ &\leq M_{gap}\cdot|D|\cdot c(\mathcal{F}_{\text{OPT}}). \end{aligned} \tag{39}$$

Finally, we prove that N-DLFC builds a light-forest whose cost is no more than $|D|\cdot M_{gap}$ times of the optimal one. ■

According to the proof above, the approximate ratio of N-DLFC is $M_{gap}\cdot|D|$. As we only consider BPSK, QPSK, 8-QAM and 16-QAM in this work, we have $m = 1$, 2, 3 and 4. Then, $M_{gap} = 4$, and the approximation ratio would be $4\cdot|D|$ in the simulations discussed in the next section.

VI. Performance Evaluation

In this section, we use extensive numerical simulations to evaluate the proposed ISa-RMSA algorithms.

A. Simulation Setup

The simulations use four topologies as shown in Fig. 5, *i.e.*, a simple six-node topology, NSFNET, Deutsche Telekom European network (DT), and US backbone topology (USB). We assume that the EON is deployed in C-band, *i.e.*, each fiber link has a total bandwidth of $B = 4.475$ THz. Using Eq. (1), we get the total number of FS' on a link as $F = 358$. For each multicast request, its source and destinations are randomly chosen and the average and maximum numbers of destinations per request are 3 and 13, respectively. The capacity requirement of each request is uniformly distributed within $[12.5, 125]$ Gb/s. The guard-band for each light-tree is set as $g_b = 1$ FS, and the reduction factor of transmission reach (*i.e.*, α in Eq. (3)) has its value chosen from $[0, 0.2]$. Note that, even though multicast and unicast signals would have the same transmission reach with $\alpha = 0$, *Theorem* 2 in Section IV-C is still valid since its proof is irrelevant to α. Specifically, in this case, the modulation-level or the spectrum usage of a light-tree is still determined by its longest branch. Hence, serving a multicast request with a light-forest might still be more beneficial than with a single light-tree. We consider both static planning and dynamic provisioning.

The simulation scenario of static network planning is pretty straightforward, *i.e.*, we first randomly generate a set of 100 multicast requests and then accommodate them in the EON by the algorithms. While for the scenario of dynamic network provisioning, we use discrete-time simulations. Specifically, at the beginning of each time slot, we release the spectrum resources of expired multicast requests. Then, we generate new requests according to the Poisson traffic model, *i.e.*, the average number of new requests that arrive in each time slot is λ while their average life-time is $\frac{1}{\mu}$ time slots. Hence, the traffic load can be quantified with $\frac{\lambda}{\mu}$ in Erlangs. Next, we try to serve the new requests by the algorithms, and a request will be blocked if we cannot find a feasible ISa-RMSA solution for it based on the current network status.

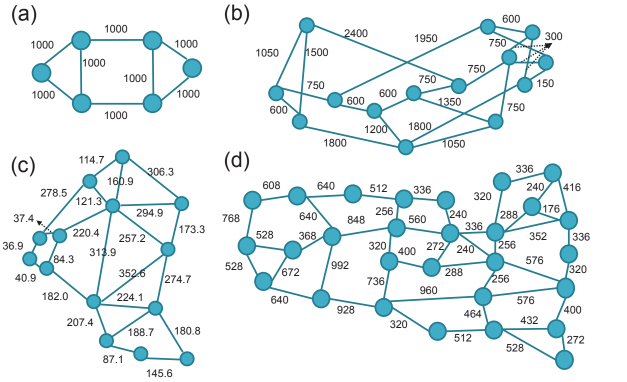


Fig. 5. Topologies used in simulations: (a) Six-node topology, (b) NSFNET, (c) Deutsche Telekom European network (DT), and (d) US backbone network (USB), with link lengths marked in kilometers.

B. Static Planning for Multicast with ISa-RMSA

We first investigate the proposed ISa-RMSA algorithms' performance in static network planning, and discuss the simulation results on the maximum index of used FS' (MSI), the average number of light-trees per request, the total bandwidth consumption in FS', and the average modulation-level per light-tree. For the algorithms that use pre-calculated light-trees (*i.e.*, N-LT-DP and B-LT-DP), we pre-calculate the light-trees with two multicast routing algorithms, *i.e.*, the shortest-path tree (SPT) and minimum spanning tree (MST). For each data point, the simulations average the results from 5 different request sets, each including 100 randomly-generated requests to ensure sufficient statistical accuracy. Note that, in static network planning, all the requests are served with light-forests, *i.e.*, there is no request blocking.

1) Performance for Different Network Topologies: Fig. 6(a) shows the results on MSI for different topologies using $\alpha = 0.12$. First of all, we observe that N-DLFC obtains the smallest MSI in all the four topologies, except for DT where its performance on MSI is similar to that of N-LT-DP (MST) and B-LT-DP (MST). For the two algorithms that use pre-calculated trees, *i.e.*, N-LT-DP and B-LT-DP, those that use MST provide smaller MSI than those with SPT in all the four topologies. We also notice that when we keep the tree pre-calculation scheme the same, B-LT-DP only achieves slightly better performance on MSI than N-LT-DP. While for the small-scale topologies, *i.e.*, six-node and DT, their results are the same. We believe that this is because the algorithms try to minimize MSI together with the average number of light-trees per light-forest (*i.e.*, the average number of BV-Ts used for each request). Specifically, when modifying a large light-tree to a light-forest, both algorithms will stop when all the light-trees become feasible ones in terms of QoT requirement. Therefore, when the topology is relatively small (*e.g.*, the case of six-node topology) or the link lengths are relatively short (*e.g.*, the case of DT topology), most of the requests would be served with light-trees instead of light-forests, and thus there would be no difference between N-LT-DP and B-LT-DP.

The results in Figs. 6(b) and 6(c) indicate that among all the proposed algorithms, N-DLFC builds light-forests with the most light-trees on average while the average modulation-level per light-tree is also the highest. Note that, the number of light-trees per light-forest indicates the usage of BV-Ts per request. The results also confirm our above analysis that N-LT-DP and B-LT-DP will use less light-trees per light-forest than N-DLFC because they will stop to divide the light-trees when all of them become feasible ones.

Table IV compares the results on total and guard-band FS' used by the algorithms for different topologies. We notice that N-DLFC achieves the lowest total FS' usage in NSFNET, six-node and USB topologies among all the algorithms. This is because when constructing the light-trees in each light-forest, N-DLFC always tries to cause the smallest bandwidth consumption increase and hence reduces the total used FS'. B-LT-DP uses less total FS' than N-LT-DP in NSFNET and USB topologies, while their performance on total FS usage is the same in DT and six-node topologies for the same reason explained above. For N-LT-DP and B-LT-DP, those with MST still outperform their counterparts with SPT in terms of total used FS' in all the topologies. It is interesting to notice that the total used FS' from N-LT-DP (MST) and B-LT-DP (MST) in DT are even less than those from N-DLFC. This is because DT has relatively short link lengths and thus almost all of the multicast requests can be served by using light-trees with high modulation-levels, which helps reduce the total FS usage.

In Table IV, the results on guard-band FS' and their percentages in the total used FS' suggest that our light-forest based provisioning schemes would not cause significant guard-band overheads. This can be explained as follows. As the bandwidth capacity of each multicast request is within $[12.5, 125]$ Gb/s in the simulations, the average capacity would be 68.75 Gb/s, which corresponds to a requirement of 6, 3, 2, and 2 FS' for BPSK, QPSK, 8-QAM, and 16-QAM, respectively. Since we have $g_b = 1$ FS, the percentage of guard-band FS' in total FS' ranges within $[14.3\%, 33.3\%]$ for this average case even when the request is served with a single light-tree. Also, we observe that the percentages of guard-band usage in DT is much higher than those in other topologies. This is still because DT has the shortest average link length among all the topologies, *i.e.*, the requests in it can use the highest modulation-level (*i.e.*, the least traffic FS') on average.

2) Impacts of Reduction Factor α: Next, we perform simulations with different values of the transmission reach reduction factor α to investigate its impacts on the algorithms' performance. This time, we only consider the NSFNET topology since it has a relatively long average link length and the impacts of α can be examined easily. In Fig. 7(a), we observe that with the increase of α, the results on MSI from all the algorithms increase slightly. This is because a larger α means that the multicast transmission reaches become shorter for all the modulation-levels, which will push the algorithms to use lower modulation-levels and thus more FS' for the requests. The results on total FS usage in Table V actually confirm this explanation. Meanwhile, the reduction of transmission reaches makes more light-trees become infeasible in terms of QoT requirement, which is the reason why Fig. 7(b) shows that the average number of light-trees per request increases with α. For the same reason, the results on guard-band FS' in Table V also increase with α slightly. Meanwhile, as expected, the average modulation-level per light-tree decreases with α slightly, as shown in Fig. 7(c). Therefore, we can conclude that changing α from 0 to 0.2 will not impact the performance of the proposed algorithms significantly and the general trend of

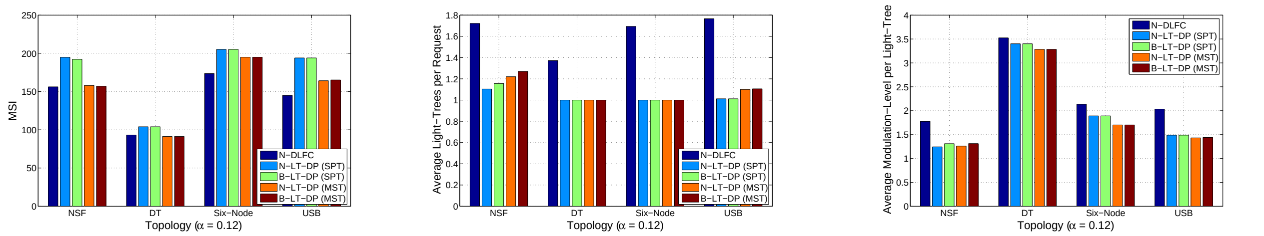
TABLE IV
TOTAL AND GUARD-BAND FS' IN STATIC PLANNING ($\alpha = 0.12$).

Topology	Algorithm	Total	Guardband	Percentage
NSFNET	N-DLFC	2848.0	523.2	18.37%
	N-LT-DP (SPT)	3427.4	548.6	16.01%
	B-LT-DP (SPT)	3350.8	539.4	16.10%
	N-LT-DP (MST)	3108.4	499.2	16.06%
	B-LT-DP (MST)	3049.4	494.0	16.20%
DT	N-DLFC	1609.4	541.2	33.63%
	N-LT-DP (SPT)	1665.4	540.4	32.45%
	B-LT-DP (SPT)	1665.4	540.4	32.45%
	N-LT-DP (MST)	1555.6	492.8	31.68%
	B-LT-DP (MST)	1555.6	492.8	31.68%
6-Node	N-DLFC	1543.0	365.8	23.71%
	N-LT-DP (SPT)	1661.6	366.0	22.03%
	B-LT-DP (SPT)	1661.6	366.0	22.03%
	N-LT-DP (MST)	1694.2	337.2	19.90%
	B-LT-DP (MST)	1694.2	337.2	19.90%
USB	N-DLFC	4059.0	829.0	20.42%
	N-LT-DP (SPT)	4754.4	823.6	17.32%
	B-LT-DP (SPT)	4752.8	823.0	17.32%
	N-LT-DP (MST)	4425.2	758.6	17.14%
	B-LT-DP (MST)	4415.0	757.0	17.15%

Finally, with the simulation results in static network planning, we can draw the following two conclusions: 1) N-DLFC generally performs the best in terms of MSI and total used FS' but the performance gap between it and N-LT-DP or B-LT-DP can vary for different topologies. Specifically, if the topology has very short link lengths as those in DT, the performance gap will vanish due to the fact that modifying a light-tree to a light-forest would not be necessary for most of the requests. 2) For the multicast provisioning with ISa-RMSA, there is a performance tradeoff between spectrum utilization and cost from BV-Ts. In general, N-DLFC is in favor of spectrum utilization while N-LT-DP and B-LT-DP can reduce the cost from BV-Ts by using a smaller number of light-trees per request. Therefore, for the topologies such as DT, we should use N-LT-DP (MST) or B-LT-DP (MST), since they result in lower cost from BV-Ts while their performance on spectrum utilization are similar to that of N-DLFC.

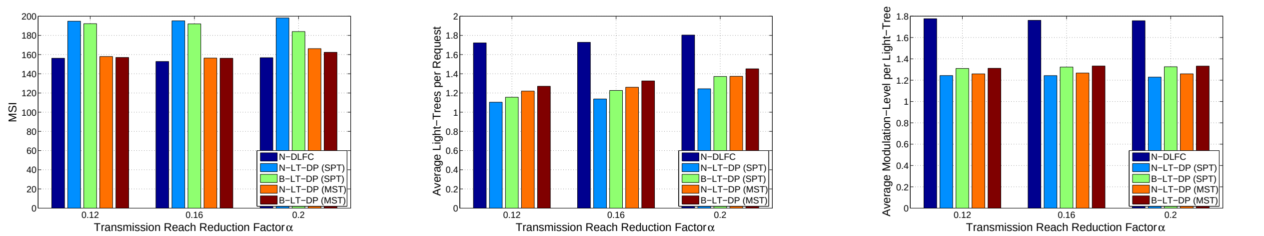
C. Dynamic Provisioning for Multicast with ISa-RMSA

We also evaluate the performance of the proposed ISa-RMSA algorithms in dynamic network provisioning. Since the



(a) MSI (b) Average number of light-trees per request (c) Average modulation-level per light-tree

Fig. 6. Simulation results of static network planning with $\alpha = 0.12$.



(a) MSI (b) Average number of light-trees per request (c) Average modulation-level per light-tree

Fig. 7. Simulation results of static network planning for NSFNET topology.

TABLE V
TOTAL FS' AND GUARD-BAND FS' WITH DIFFERENT α IN NSFNET

α	Algorithm	Total	Guardband	Percentage
0	N-DLFC	2762.0	513.6	18.60%
	N-LT-DP (SPT)	3368.8	543.2	16.12%
	B-LT-DP (SPT)	3320.4	537.6	16.19%
	N-LT-DP (MST)	2950.6	480.4	16.28%
	B-LT-DP (MST)	2922.6	479.0	16.39%
0.12	N-DLFC	2848.0	523.2	18.37%
	N-LT-DP (SPT)	3427.4	548.6	16.01%
	B-LT-DP (SPT)	3350.8	539.4	16.10%
	N-LT-DP (MST)	3108.4	499.2	16.06%
	B-LT-DP (MST)	3049.4	494.0	16.20%
0.16	N-DLFC	2885.2	528.0	18.30%
	N-LT-DP (SPT)	3446.2	552.0	16.02%
	B-LT-DP (SPT)	3361.6	543.2	16.16%
	N-LT-DP (MST)	3125.6	504.8	16.15%
	B-LT-DP (MST)	3066.4	500.0	16.31%
0.2	N-DLFC	2931.6	535.8	18.28%
	N-LT-DP (SPT)	3502.0	561.8	16.04%
	B-LT-DP (SPT)	3374.6	548.8	16.26%
	N-LT-DP (MST)	3206.4	519.4	16.20%
	B-LT-DP (MST)	3126.0	511.8	16.37%

six-node topology is relatively small and cannot accommodate many multicast requests, we only perform simulations with the DT, NSFNET and USB topologies. Note that, since the simulation results of static network planning already indicate that for N-LT-DP and B-LT-DP, the MST-based ones always perform better than their SPT-based counterparts, we only simulate the MST-based ones in dynamic network provisioning. Moreover, we consider the fragmentation-aware scheme in Subsection V-D, and for each of the proposed algorithms, it incorporates the fragmentation-aware scheme if we add "F-" in front of its name. For instance, F-N-DLFC is the counterpart of N-DLFC that incorporates the fragmentation-aware scheme.

Fig. 8 shows the results on blocking probability with $\alpha = 0.12$. It can be seen that as compared with their counterparts without the fragmentation-aware scheme, the fragmentation-aware ISa-RMSA algorithms achieve lower blocking probability in all the three topologies, which confirms the effectiveness of the fragmentation-aware scheme. Note that, as in the fragmentation-aware algorithms, we need to update the links' fragmentation ratios and find the weighted shortest paths based on them for each request, their time complexities are also higher. The results in Fig. 8(a) indicate that with the DT topology, N-LT-DP and B-LT-DP perform the same in terms of blocking probability, no matter whether the fragmentation-aware scheme is incorporated or not. Again, this is because when the link lengths in the topology are relatively short, most of the multicast requests would be served with light-trees instead of light-forests, and thus there would be no difference between N-LT-DP and B-LT-DP. We also notice that F-B-LT-DP and F-N-LT-DP achieve the best blocking performance with the DT topology, and their results on blocking probability are slightly better than those of F-N-DLFC. Hence, we can conclude that for the topologies such as DT, we should use F-N-LT-DP or F-B-LT-DP instead of F-N-DLFC, which matches our finding in the static network planning.

Figs. 8(b) and 8(c) show the results on blocking probability for the NSFNET and USB topologies, respectively. Basically, for the algorithms without the fragmentation-aware scheme, N-DLFC achieves the best blocking performance, while the blocking probability from N-LT-DP is the highest, which also matches our finding in the static network planning. Among all the proposed algorithms, F-N-DLFC achieves the best blocking performance. The results on blocking probability with $\alpha = 0$ and 0.2 are plotted in Figs. 9 and 10, respectively, which exhibit the similar trends as those in Fig. 8.

VII. Conclusions

We have investigated impairment- and splitting-aware RMSA (ISa-RMSA) for multicast provisioning in EONs. We first studied the procedure of adaptive modulation selection for a light-tree, and formulated the mathematical problem of ISa-RMSA for all-optical multicast in EONs. Then, we leveraged the flexibility of routing structures and discussed both light-tree and light-forest in ISa-RMSA. Specifically, we explored the optimal light-tree structure and also defined the minimum light-forest problem for optimizing a light-forest. Our study indicated that for ISa-RMSA, the light-forest based approach could use less bandwidth than the light-tree based one, while still satisfying the QoT requirement. Finally, we designed several time-efficient ISa-RMSA algorithms, and proved that N-DLFC can solve the minimum light-forest problem with a fixed approximation ratio.

Our simulation results indicated that for both static planning and dynamic provisioning, N-DLFC generally performed the best since it could build the minimum light-forests but the performance gap between it and N-LT-DP or B-LT-DP could vary for different topologies. Specifically, if the topology has very short link lengths as those in DT, the performance gap would vanish because modifying a light-tree to a light-forest would not be necessary for most of the requests. Moreover, since N-LT-DP and B-LT-DP tended to use a smaller number of light-trees per request, we should use them instead of N-DLFC for the topologies such as DT. Nevertheless, if the topology has a relatively large scale and long link lengths as the cases of NSFNET and USB, we should use N-DLFC because it could balance the performance tradeoff between spectrum utilization and cost from BV-Ts better. Finally, for dynamic provisioning, incorporating the fragmentation-aware scheme in the ISa-RMSA algorithm would be helpful since it could further reduce the blocking probability.

Acknowledgments

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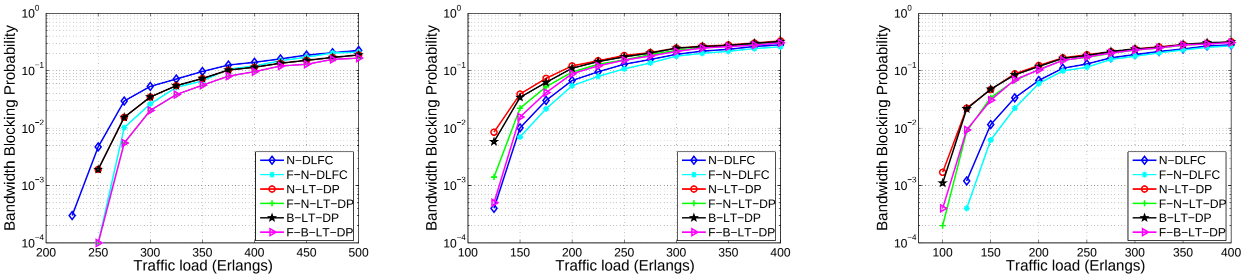
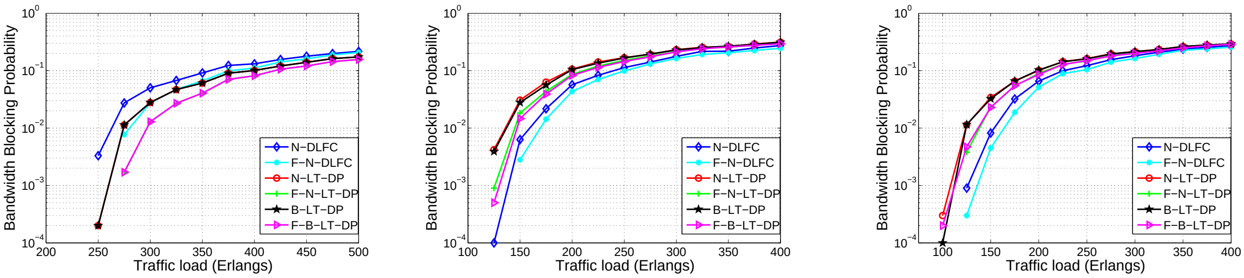


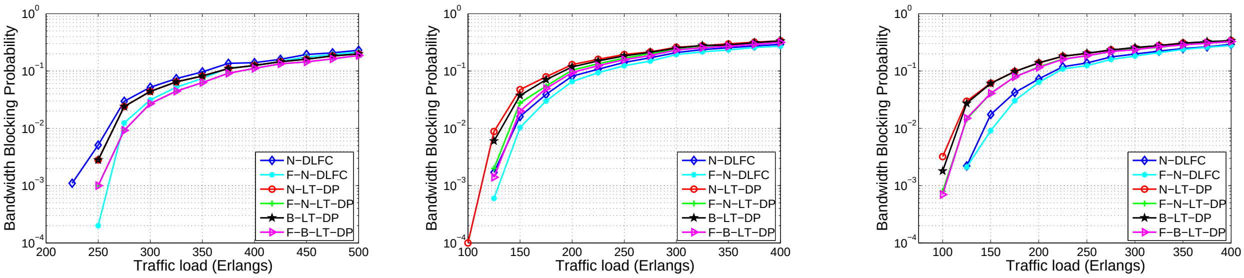
(a) DT (b) NSFNET (c) USB

Fig. 8. Blocking probability from ISa-RMSA in dynamic provisioning ($\alpha = 0.12$).



(a) DT (b) NSFNET (c) USB

Fig. 9. Blocking probability from ISa-RMSA in dynamic provisioning ($\alpha = 0$).



(a) DT (b) NSFNET (c) USB

Fig. 10. Blocking probability from ISa-RMSA in dynamic provisioning ($\alpha = 0.2$).

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