Design Integrated RSA for Multicast in Elastic Optical Networks with a Layered Approach

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***Abstract*—In this paper, we incorporate a layered approach to design integrated multicast-capable routing and spectrum assignment (MC-RSA) algorithms for achieving efficient all-optical multicasting in spectrum-sliced elastic optical networks (EONs), which are based on the optical orthogonal frequency-division multiplexing (O-OFDM) technology. For each multicast request, the proposed algorithms decompose the physical topology into several layered auxiliary graphs according to the network spectrum utilization. Then, based on the request's bandwidth requirement, we select a proper layer and calculate a multicast light-tree within it. With these procedures, the routing and spectrum assignment (RSA) for each multicast request is done in an integrated way. We evaluate the proposed algorithms in simulations of static network planning and dynamic network provisioning. The simulation results demonstrate that compared to the existing MC-RSA algorithms, our approaches achieve more efficient network planning in terms of spectrum utilization, and provide lower blocking probabilities in network provisioning.**

***Index Terms*—Optical orthogonal frequency-division multiplexing (O-OFDM), Elastic optical networks (EONs), All-optical multicasting, Routing and spectrum assignments (RSA), Layered graph model**

I. INTRODUCTION

Nowadays, the popularity of bandwidth-hungry applications has driven the Internet traffic to grow exponentially. This traffic growth has stimulated intensive research activities on fiber-optic technologies, for scaling backbone networks with the rising trend of bandwidth requirement. Recent advances on the optical orthogonal frequency-division multiplexing (O-OFDM) technology [1] have demonstrated high bandwidth efficiency and flexible bandwidth allocation [2]. An O-OFDM transponder grooms the capacities of several contiguous narrow-band subcarrier frequency channels (slots) to support a high-speed connection request, and can tailor the bandwidth allocation by adjusting the number of assigned frequency slots. Meanwhile, thanks to the technology advances in liquid crystal-on-silicon wavelength-selective switch (LCOS-WSS), a switching node can achieve the switching granularity at 12.5 GHz or less [3]. Since O-OFDM networks realize more flexible bandwidth resource management than fixed-grid wavelength-division multiplexing (WDM) networks, people tended to refer them as spectrum-sliced elastic optical networks (EONs) [2].

Together with its advantages, O-OFDM also brings challenges for future optical networks. For instance, evolved from the famous routing and wavelength assignment (RWA) problem in WDM networks, routing and spectrum assignment (RSA) in EONs needs to manipulate blocks of contiguous frequency slots instead of independent wavelength channels. Hence, more sophisticated network planning and provisioning procedures are required for high-efficient operations. To address these challenges, numerous RSA algorithms have already been proposed in literatures [4–10]. However, the RSA algorithms for all-optical multicasting over EONs are still under-explored.

It is known that multicast is widely used to support applications such as teleconference, IP television, stock exchanges and etc, and it is making important contributions to the Internet traffic. Moreover, there recently has been a growing demand for supporting scientific applications that can transfer Petabyte-scale data to numbers of geographically dispersed users [11]. Compared to conventional IP multicasting, all-optical multicasting can provide a more transparent and energy-efficient solution due to the reason that repeated optical-electrical-optical (O/E/O) conversions can be reduced to the maximum extent [12]. For all-optical multicasting in WDM networks, previous works have investigated multicast-capable RWA in [13–15]. Since O-OFDM achieves more flexible bandwidth management in the optical layer, we expect future EONs to provide more efficient support to all-optical multicasting, especially for those induced by the scientific applications whose traffic can have relatively large variations. Therefore, it is desired and important to have efficient multicast-capable RSA (MC-RSA) algorithms for EONs. Recently, Wang *et al.* performed a performance analysis of two MC-RSA algorithms that could support all-optical multicasting in EONs [16]. The MC-RSA algorithms were designed to use routing strategies based on either the shortest path tree (SPT) or the minimum spanning tree (MST) algorithms [17], together with first-fit spectrum assignment. However, as we will show later, the solutions they obtained have the drawback of low spectrum utilization due to spectrum fragmentation [18].

In this paper, we incorporate a layered approach to design integrated MC-RSA algorithms for serving multicast requests efficiently in EONs. For each multicast request, the proposed algorithms decompose the physical topology into several layered auxiliary graphs according to the network spectrum utilization. Then, based on the request's bandwidth requirement, we select a proper layer and calculate a multicast light-tree within it. With these procedures, the routing and

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spectrum assignment for each multicast request is done in an integrated way. The rest of the paper is organized as follows. Section II formulates the problem of serving multicast request all-optically in EONs, and describes the design constraints and objective. In Section III, we discuss the proposed integrated MC-RSA algorithms. Then, the performance evaluation is shown in Section IV. Finally, Section V summarizes the paper.

II. PROBLEM FORMULATION

In this section, we formulate the multicast-capable routing and spectrum assignment (MC-RSA) problem, including the design constraints and objective.

A. Design Constraints

In this paper, we use a directed graph $G(V, E)$ to represent the physical topology of an EON, where V and E denote the sets of nodes and fiber links, respectively. We define a multicast request as $R(s, D, n)$, where $s \in V$ is the source node, $D \subseteq V \setminus s$ is the set of destinations, and n is the number of frequency slots (FS's) it requires (including the guard-band). We assume that there are no spectrum convertors in the network, and all optical cross-connects (OXCs) in the nodes are multicast-capable with splitter-and-delivery (SAD) switches [19]. With the SAD switches, we incorporate the same-spectrum all-optical multicasting scheme, similar to the same-wavelength scheme in WDM networks [20].

For a multicast request $R(s, D, n)$, the MC-RSA needs to find a light-tree $\mathcal{T}$ that roots at s and can reach all destinations in D, and to assign n FS's on each link $e \in \mathcal{T}$ under the spectrum continuity and spectrum non-overlapping constraints. We assume that there are F FS's on each fiber link, and define a spectrum-usage bit-mask $b_e[1 \dots F]$ for each link $e \in E$. We set $b_e[j] = 1$, if the j^{th} FS on link e is taken, otherwise, $b_e[j] = 0$. When there are multiple multicast requests, we assign a unique index i to each request and denote it as R_i. If we define $\mathcal{T}_i$ and $\mathcal{F}_{i,e}$ as the light-tree of and set of assigned FS's on link $e \in \mathcal{T}_i$ for R_i, respectively, then the constraints of MC-RSA can be described as follows,

- Spectrum continuity constraint:

$$\mathcal{F}_{i,e_1} = \mathcal{F}_{i,e_2}, \qquad \forall e_1, e_2 \in \mathcal{T}_i. \tag{1}$$

  Eq. (1) ensures that all links on the light-tree of R_i use the same set of FS's (*i.e.*, all-optical multicasting).
- Spectrum non-overlapping constraint[1]:

$$\mathcal{F}_{i_1,e} \cap \mathcal{F}_{i_2,e} = \emptyset, \quad \text{if } e \in \mathcal{T}_{i_1} \cap \mathcal{T}_{i_2}, \quad \forall i_1 \neq i_2. \tag{2}$$

  Eq. (2) ensures that any two requests whose light-trees have common links do not use the same FS's.
- Spectrum contiguous and rate constraints:

$$FS_{j_i+\nu,e} \in \mathcal{F}_{i,e}, \quad \forall e \in E, \nu \in \{0, 1, \cdots, n_i - 1\}. \tag{3}$$

  where n_i is the number of FS's required by R_i, $FS_{j,e}$ denotes the j^{th} FS on link $e \in E$, and j_i is the index of the starting FS assigned to R_i. Eq. (3) ensures that there are n_i FS's allocated to R_i and they are contiguous in the spectrum domain.

B. Objective

1) Static Network Planning: In static network planning, all the multicast requests are known *a priori*, and they all have to be accommodated in the EON simultaneously through MC-RSA, *i.e.*, we do not consider request blocking in this case. In order to improve the spectral efficiency of network planning, we define the objective similar to those of static RSA for unicast requests [4, 6], as to minimize the maximum index of the used FS's on all links after serving all the requests,

$$Minimize \qquad \xi = \max_{e \in E} \{j : b_e[j] = 1\}, \tag{4}$$

where b_e is the spectrum-usage bit-mask of link e, after serving all the requests.

2) Dynamic Network Provisioning: In dynamic network provisioning, the requests are associated with two additional parameters, arrival time and holding period, since they are time-variant and can arrive and leave on-the-fly. Thus, we denote a dynamic multicast request as $R(\tau, h, s, D, n)$, where τ and h are the arrival time and holding period, respectively, while s, D, n share the same definitions as those of static request. For dynamic MC-RSA, we aim at minimizing the blocking probability,

$$Minimize \qquad p_b = \lim_{T \to \infty} \frac{N_b(T)}{N_t(T)}, \tag{5}$$

where $N_b(T)$ and $N_t(T)$ are the numbers of blocked and total requests in the time duration $[0, T]$.

III. INTEGRATED MC-RSA WITH A LAYERED APPROACH

In this section, we explain the detailed procedures of the proposed integrated MC-RSA algorithms.

A. Layered Auxiliary Graphs

The proposed MC-RSA algorithm achieves light-tree selection (*i.e.*, routing) and spectrum assignment in one integrated step with the assistance of layered auxiliary graphs. Given a request $R(s, D, n)$, the proposed algorithm first decomposes the physical topology $G(V, E)$ into a few layered auxiliary graphs, where the k^{th} layer graph, $G^k(V^k, E^k)$, is constructed as follows,

$$V^k = V \tag{6}$$

$$E^k = \{e : \sum_{j=k}^{k+n-1} b_e[j] = 0, e \in E\} \tag{7}$$

More specifically, to construct $G^k(V^k, E^k)$, we scan the spectrum utilization of the network, and insert a direct link $e^k = (u^k, v^k)$ in $G^k(V^k, E^k)$ if starting from the k^{th} FS, there are n available contiguous FS's on $e = (u, v)$ in $G(V, E)$. Hence, if we can obtain a light-tree for $s^k \to D^k$ in $G^k(V^k, E^k)$, the multicast request can be served with the light-tree, using the k^{th} to $(k + n - 1)^{th}$ FS's in $G(V, E)$.

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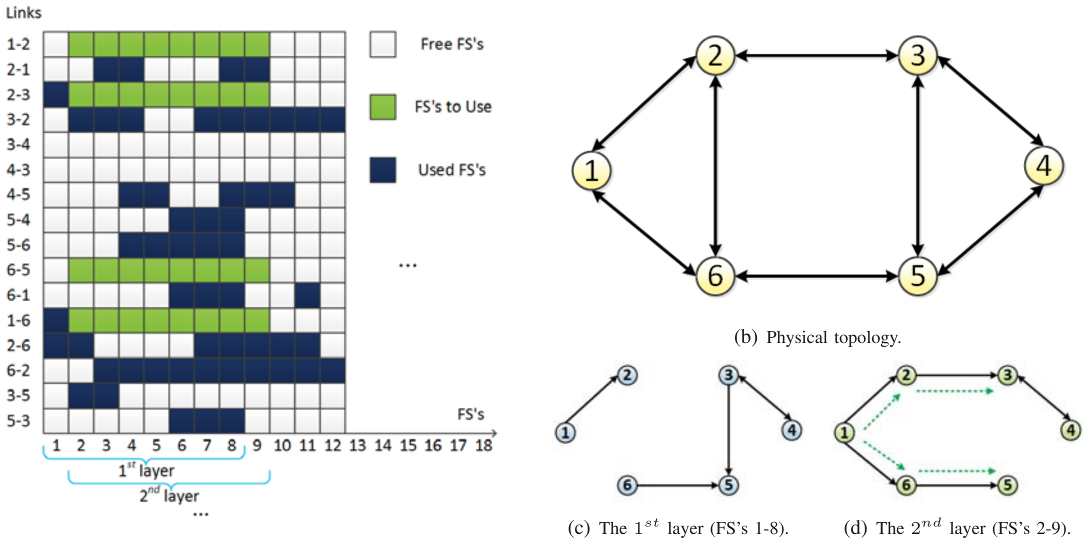
[1] Note that this constraint does not apply when R_{i_1} and R_{i_2} are non-overlapping in the time domain in dynamic network operations.



Links	1	2	3	4	5	6	7	8	9	10	11	12
1-2	□	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	□	□	□
2-1	□	□	■	■	□	□	□	■	■	□	□	□
2-3	■	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	□	□	□
3-2	□	■	■	■	□	□	■	■	■	■	■	■
3-4	□	□	□	□	□	□	□	□	□	□	□	□
4-3	□	□	□	□	□	□	□	□	□	□	□	□
4-5	□	□	□	■	■	□	□	■	■	■	□	□
5-4	□	□	□	□	□	■	■	■	□	□	□	□
5-6	□	□	□	■	■	■	■	■	□	□	□	□
6-5	□	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	□	□	□
6-1	□	□	□	□	□	■	■	■	□	□	■	□
1-6	■	🟩	🟩	🟩	🟩	🟩	🟩	🟩	🟩	□	□	□
2-6	■	■	□	□	□	□	■	■	■	■	■	□
6-2	□	□	■	■	■	■	■	■	■	■	■	■
3-5	□	■	■	□	□	□	□	□	□	□	□	□
5-3	□	□	□	□	□	■	■	■	□	□	□	□

(a) Network spectrum utilization.

(b) Physical topology.

(c) The 1^{st} layer (FS's 1-8).

(d) The 2^{nd} layer (FS's 2-9).

Fig. 1. An example of constructing layered auxiliary graph for MC-RSA.

Fig. 1 shows an intuitive example of how to construct the layered auxiliary graphs. The physical topology is shown in Fig. 1(b). We have a multicast request as $R(1,\{3,5,6\},8)$ to serve. Based on the network spectrum utilization in Fig. 1(a), we construct layered auxiliary graphs as illustrated in Fig. 1(c) (1^{st} layer) and Fig. 1(d) (2^{nd} layer). Since we can obtain a feasible light-tree in the 2^{nd} layer, the multicast request is served with the light-tree represented by the dash line in Fig. 1(d), using the slots in the 2^{nd} layer.

B. Integrated MC-RSA Algorithms

Algorithm 1 illustrates the procedures of the proposed integrated MC-RSA algorithm with the layered approach. *Lines* 4-9 are for constructing a layered auxiliary graph. *Line* 11 is for calculating the light-tree for the multicast request, which should be rooted at s and covers all destinations in D. In the context of this work, we apply two algorithms to obtain the light-tree in $G^k(V^k,E^k)$, the shortest-path tree (SPT) and the minimum spanning tree (MST) [17]. The SPT algorithm constructs a light-tree by first finding the shortest path from the s^k to each destination in D^k and then merging the paths together. For the MST algorithm, we calculate a light-tree according to the algorithm proposed in [17].

C. Complexity Analysis

With the procedures illustrated in *Algorithm* 1, we refer the integrated MC-RSA algorithm with the layered SPT approach as MC-RSA-LSPT, and refer the one with the layered MST as MC-RSA-LMST. If we use the Fibonacci-heap data structure, the time complexity of the Dijkstra's algorithm is $\mathcal{O}(|E|+|V|\log|V|)$ according to [21]. Then, the time complexity of calculating a light-tree with SPT is $\mathcal{O}((|E|+|V|\log|V|)|D|)$. Meanwhile, the time complexity of the MST algorithm is $\mathcal{O}((|D|+1)|V|^2)$ according to [17]. According to [22], the time complexity for checking whether s can reach all destinations in D is $\mathcal{O}(|E|+|V|)$ with breadth-first-search (BFS). Therefore, for the worst case, the time complexity of MC-RSA-LSPT is $\mathcal{O}((|E|+|V|)(F-n)+(|E|+|V|\log|V|)|D|)$, and that of MC-RSA-LMST is $\mathcal{O}((|E|+|V|)(F-n)+(|D|+1)|V|^2)$.

Algorithm 1: Integrated MC-RSA Algorithm with the Layered Approach

```
input : The physical topology $G(V,E)$, a multicast
        request $R(s,D,n)$, the maximum number of
        FS's on each link $F$, and the spectrum-usage
        bit-masks $\{b_e, e \in E\}$.
output: Light-tree $\mathcal{T}$ and allocated FS's $\mathcal{F}$ for the
        multicast request.
1  $\mathcal{T} \leftarrow \emptyset$;
2  $\mathcal{F} \leftarrow \emptyset$;
3  for $k = 1$ to $F - n + 1$ do
4      insert all $v \in V$ in $G^k(V^k, E^k)$ as $v^k$;
5      for all links $e \in E$ do
6          if $sum(b_e[k \dots (k+n-1)]) = 0$ then
7              insert $e$ in $G^k(V^k, E^k)$ as $e^k$;
8          end
9      end
10     if $s^k$ can reach all destinations in $D^k$ then
11         apply SPT or MST algorithm in $G^k(V^k, E^k)$
           for $\mathcal{T} = s^k \rightarrow D^k$;
12         $\mathcal{F} \leftarrow \{FS_k, \dots, FS_{k+n-1}\}$;
13         break;
14     end
15 end
16 return $\mathcal{T}$ and $\mathcal{F}$;
```

D. Static Network Planning with Integrated MC-RSA

As explained in Section II-B, we know all multicast requests are accommodated simultaneously in static network planning. *Algorithm* 2 shows how to incorporate the integrated MC-RSA algorithms for static network planning.

Algorithm 2: Static Network Planning with Integrated MC-RSA

```
input : The physical topology $G(V, E)$, the multicast
        requests $\mathcal{R} = \{R_i(s_i, D_i, n_i), i \in \mathbb{Z}^+\}$, and the
        spectrum-usage bit-masks $\{b_e, e \in E\}$.
output: RSA solutions of $\mathcal{R}$ with light-trees $\mathcal{T}_i$ and
        assigned FS's $\mathcal{F}_i$.

initialize $\{b_e, e \in E\}$;
sort requests in $\mathcal{R}$ in descending order of $n_i$;
for all $R_i \in \mathcal{R}$ do
    apply Algorithm 1 to $R_i$ and $G(V, E)$;
    get $\mathcal{T}_i$ and $\mathcal{F}_i$ for $R_i$;
    update $\{b_e, e \in E\}$;
end
```

E. Dynamic Network Provisioning with Integrated MC-RSA

In dynamic network provisioning, we need to serve all requests arrived in ceratin service period at each service provision time, while the future requests are unknown. When there is insufficient network resource to provision a request, the request is blocked. Note that we do not allow partial provisioning of a multicast request and R_i would be blocked even if we cannot serve only one destination in D_i. *Algorithm* 3 illustrates the procedures of dynamic network provisioning with the integrated MC-RSA.

IV. PERFORMANCE EVALUATION

In this section, we evaluate the proposed integrated MC-RSA algorithms in both static network planning and dynamic network provisioning. The simulations are carried out with two topologies, the 14-node NSFNET topology [23] and the 28-node US Backbone topology [24]. We assume that the capacity of a single FS is 12.5 Gb/s and the number of FS's required by each request, n_i, is uniformly distributed within $\{1, 2, ..., 10\}$. For each multicast request, we select the source s_i randomly and assume that the rest of the nodes have a fixed probability of p_{join} to join the multicast group. We set p_{join} as 0.286 and 0.143 in NSFNET and US Backbone topologies, respectively, and make the average number of multicast destinations (*i.e.*, $|D_i|$) as 3. The two algorithms proposed in [16] are used as benchmark algorithms. Since they do not incorporate the layered approach, we refer them as MC-RSA-SPT and MC-RSA-MST.

A. Static Network Planning

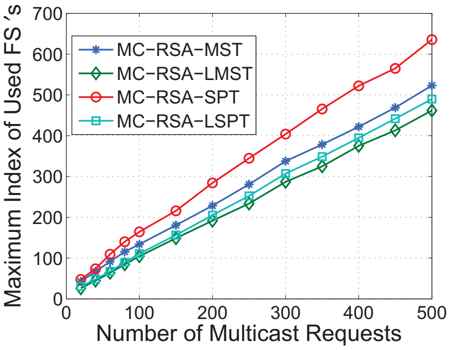
Fig. 2(a) and 3(a) show the maximum indices of the used FS's, ξ, from different MC-RSA algorithms in NSFNET and US Backbone topologies, respectively. We observe that when the number of requests is the same, the algorithms with the layered approaches always provides smaller ξ than those without the layered approaches. More specifically, compared to MC-RSA-SPT and MC-RSA-MST, MC-RSA-LSPT and MC-RSA-LMST can reduce ξ up to 45% and 40%, respectively. Between the two MC-RSA algorithms with the layered approaches, MC-RSA-LMST provides smaller ξ than MC-RSA-LSPT. This is because that compared to SPT, the MST algorithm can find more efficient light-tree for routing (*i.e.*, total link number is equal or less).

Algorithm 3: Dynamic Network Provisioning with Integrated MC-RSA

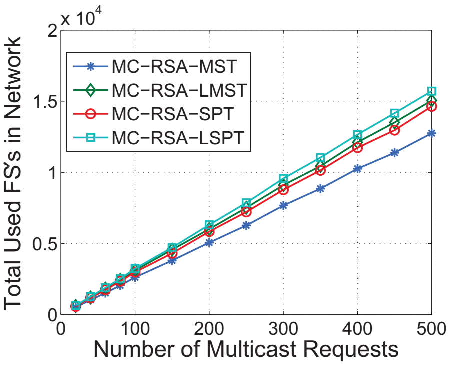
```
input : The physical topology $G(V, E)$, the maximum
        number of FS's on each link $F$, and the
        spectrum-usage bit-masks $\{b_e, e \in E\}$.

initialize $\{b_e, e \in E\}$;
while the network is operational do
    collect the requests $\mathcal{R} = \{R_i(s_i, D_i, n_i), i \in \mathbb{Z}^+\}$
    arrived in this service period;
    release the resources of the expired requests;
    wait for the service provision time;
    sort all the pending requests in $\mathcal{R}$ in descending
    order of $n_i$;
    for all the pending requests $R_i \in \mathcal{R}$ do
        apply Algorithm 1 to get $\mathcal{T}_i$ and $\mathcal{F}_i$ for request
        $R_i$;
        if $\mathcal{T}_i = \emptyset$ then
            mark $R_i$ as blocked;
        else
            update $\{b_e, e \in E\}$;
        end
    end
end
```

Fig. 2(b) and 3(b) show the total number of used FS's from different MC-RSA algorithms. Basically, if a light-tree goes across m links and we assign n FS's on each of these links, the total number of used FS's on the light-tree is $m \cdot n$. It is interesting to notice that the total number of used FS's from the MC-RSA algorithms with the layered approaches are always slightly larger than those without. This because that the MC-RSA algorithms without the layered approaches tend to use the most efficient light-trees for routing, regardless of the spectrum-usage on them. Hence, for a multicast request, MC-RSA-SPT or MC-RSA-MST can get a light-tree with smaller number of links than MC-RSA-LSPT or MC-RSA-LMST. However, since they do not consider spectrum-usage of the links during routing, MC-RSA-SPT and MC-RSA-MST can cause unbalanced load distribution in the network and generate a lot of bandwidth fragmentations [18]. Therefore, their performance on ξ is worse than that of MC-RSA-LSPT and MC-RSA-LMST. A network operator usually scales its network according to ξ from the network planning. If the



(a) Maximum index of the used FS's, ξ, from different MC-RSA algorithms.



(b) Total number of the used FS's from different MC-RSA algorithms.

Fig. 2. Simulation results of static network planning in the NSFNET topology.

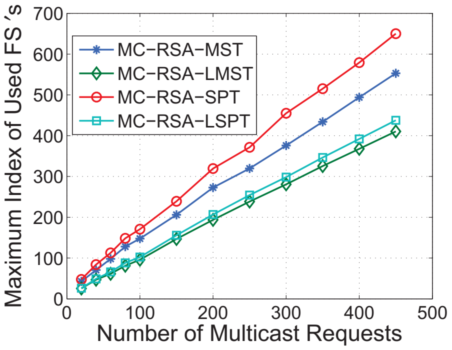
network operator needs to allocate a larger number of FS's on each fiber link while the total number of FS's that are actually used is smaller, more spectrum resources are wasted. Thus the simulation results in Fig. 2 and 3 also demonstrate that MC-RSA-LSPT and MC-RSA-LMST can significantly reduce spectrum waste.

Table I shows the average running time per multicast request of the algorithms in NSFNET topology. The simulations are carried out in Matlab R2011b environment on a computer with 3.1 GHz Intel Core i5 CPU and 4 GB RAM. As expected, MC-RSA-LSPT and MC-RSA-LMST consume more time than MC-RSA-SPT and MC-RSA-MST, due to the additional time complexity from the layered approaches. Nevertheless, the results in Table I also verify that the computational complexities of MC-RSA-LSPT and MC-RSA-LMST are still well-controlled and acceptable for practical operations. For instance, the running time per multicast request is still less than 49 msec, even for the most complicated simulation scenario that has 500 requests to serve.

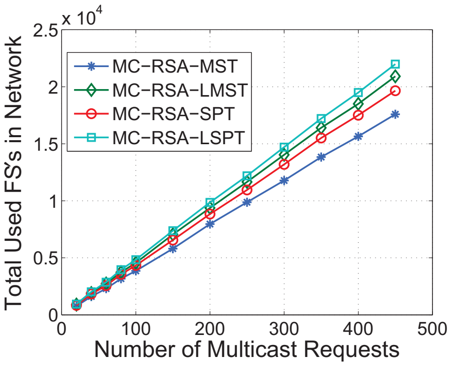
B. Dynamic Network Provisioning

In dynamic network provisioning, we assume that the network is deployed in the C-band, and there is ~ 4.475 THz spectrum on each fiber link, which corresponds to 358 FS's. The multicast requests arrive according to a Poisson process with an average arrival rate of λ requests per time unit, and the holding period of each request follows the negative exponential distribution with an average of $\frac{1}{\mu}$ time units. Hence the traffic load can be quantified with $\frac{\lambda}{\mu}$ in Erlangs.

Fig. 4 shows the simulation results on blocking probability. It can be seen that among the four algorithms, MC-RSA-LMST achieves the best performance on blocking probability. While the blocking probabilities from MC-RSA-LSPT are always lower than those from the MC-RSA-SPT, they do not have significant difference from those provided by MC-RSA-MST for the NSFNET topology. However, when the network becomes more connected as in the US Backbone



(a) Maximum index of the used FS's, ξ, from different MC-RSA algorithms.

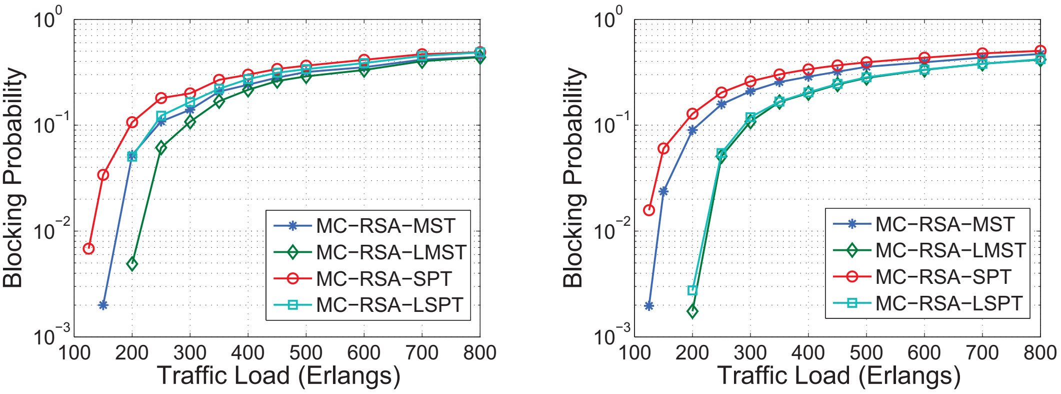


(b) Total number of the used FS's from different MC-RSA algorithms.

Fig. 3. Simulation results of static network planning in the US Backbone topology.

TABLE I
RUNNING TIME OF STATIC NETWORK PLANNING (MSEC)

# of Requests	MC-RSA- MST	LMST	SPT	LSPT
20	11.1	13.7	2.5	6.0
60	10.7	16.6	2.5	9.3
100	11.6	21.4	2.6	13.3
200	11.2	28.1	2.7	22.1
300	11.5	35.8	2.8	30.3
400	11.9	43.0	3.1	37.9
500	11.7	48.8	3.2	45.6



(a) Blocking probability in the NSFNET topology.

(b) Blocking probability in the US Backbone topology.

Fig. 4. Simulation results of dynamic network provisioning.

topology, MC-RSA-LSPT can achieve much better performance on blocking probability than MC-RSA-MST. The simulation results verify that the MC-RSA algorithms with the layered approach can also effectively reduce request blocking probability in dynamic network provisioning.

V. CONCLUSION

We incorporated a layered approach to design integrated MC-RSA algorithms for serving multicast requests efficiently in O-OFDM based EONs. The proposed algorithms were evaluated in simulations of static network planning and dynamic network provisioning. The simulation results demonstrated that compared to the existing MC-RSA algorithms, our approaches achieved more efficient network planning in terms of spectrum utilization, and provided lower blocking probabilities in network provisioning.

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